# Readiness Review Series

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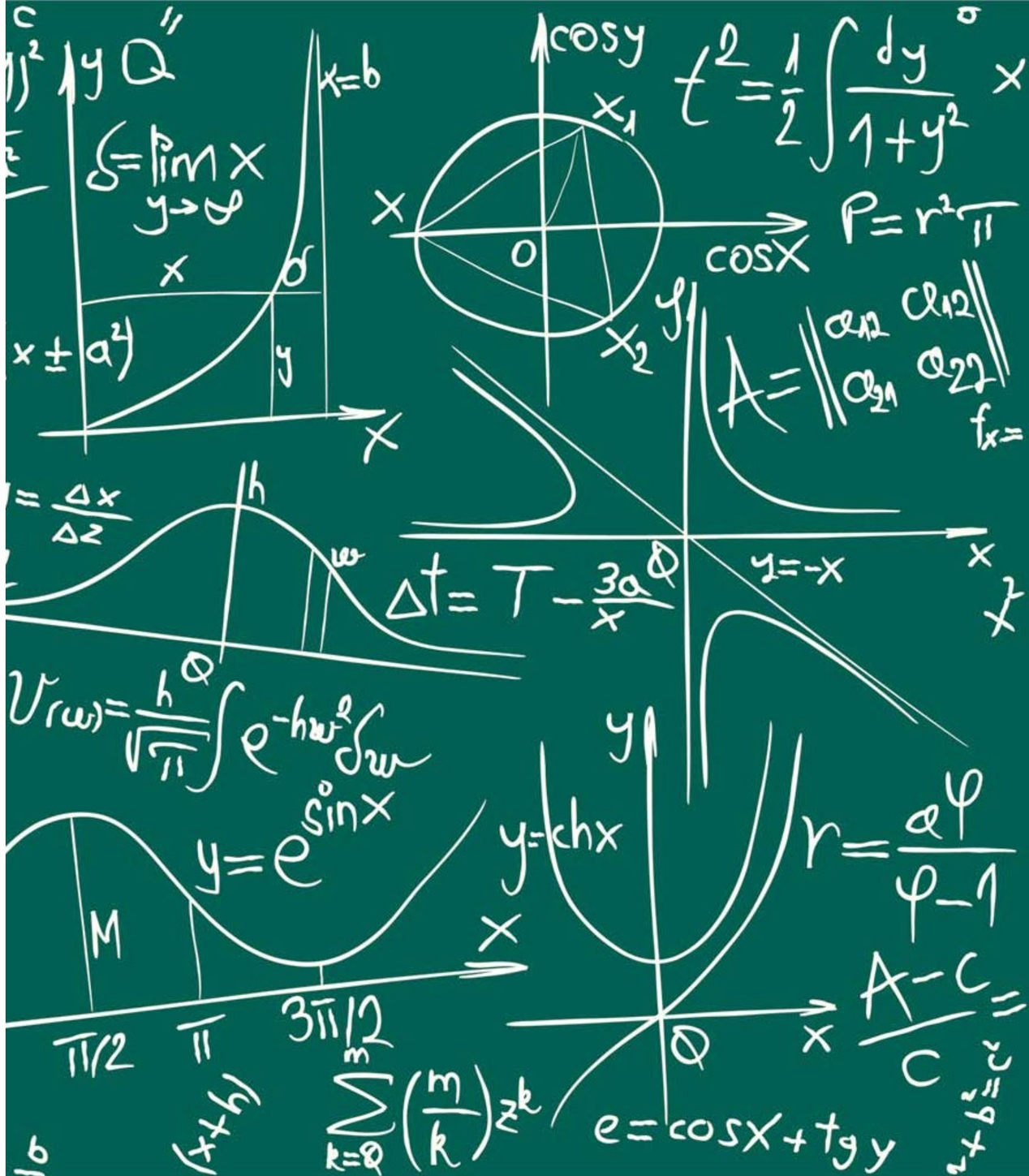

# Tips for Math Success

## Believe that your intelligence can grow.

The first battle in learning anything is believing you're capable. Cultivating a growth mindset (i.e. believing you're fully capable of mastering new material) will help immeasurably in achieving academic success.

## Practice by teaching someone else a difficult concept.

Explaining a concept to someone else will help you work through the material yourself and reinforce the information in your own mind. Once you can successfully teach someone else the concept, you know you've mastered it.

## Get tutoring weekly.

ACE, CARE, and Libraries' Learning District all offer free help for math courses. Visit tutoring.fsu.edu for more information on what you can get help with and how to access those programs.

## Memorize key formulas/theorums.

Having crucial formulas and concepts on hand will help you navigate the more complex concepts to come.

# Tips for Math Success

## Always attend class.

This is the best chance to hear extra explanations, ask questions, and gain a stronger understanding of how each concept fits into the overall subject matter.

## Start homework the day it is assigned.

The best practice is to complete homework problems without using example problems as a guide or copying answers from another source. Also, even when it is not worth points, you should focus on mastering the content of these assignments.

## Make your schedule work for you.

If you are taking 15 credits this semester, create a weekly study schedule with 25 hours of study time during the week. It's also better to create 30-minute time blocks per class throughout the week as opposed to cramming. You'll remember a lot more when exposed to the material multiple times by practicing problems 2 or 3 times outside of class each week.

## Be an active reader.

When you read your textbook, paraphrase each paragraph or section to ensure you understand. It can also help to color code your notes to help you identify what you do not understand and give you the chance to ask for clarification later.

# Tips for Math Success

## Ask for help.

Spend a little bit of time trying to resolve it yourself, but don't spin your wheels. If something doesn't make sense or you feel stuck on a problem or concept, reach out to the instructor or the TAs for guidance. Visiting your instructors regularly in office hours will help you to develop better communication channels and to master the content you do not understand.

## If you work with a tutor, make sure you have done some legwork before the tutoring session.

Make sure you know where you could use the additional help so that your tutoring session is effective and efficient.

## Work well in advance of deadlines.

Last-minute emergencies and conflicts can never be predicted. You don't want to miss out on earning points because of procrastination.

## Be comfortable being uncomfortable.

Learning takes time, and until we have mastered something, we may often lack confidence in our abilities and our knowledge. The more time you spend studying something, the more comfortable you will become with the topic. However, be patient with yourself as you are learning.

# MAC1105 – College Algebra

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# Vocabulary Review

Test yourself with these key terms.

- **Quadratic Equations:** Equations that can be rearranged into standard form
- **Determinants:** A scalar value of a function of a square matrix
- **Linear Equations:** Equation with two variables that creates a straight line on a graph
- **Conjugates:** Two pairs of binomials with the same terms where the sign between the two terms is the opposite
- **Polynomials:** An expression of the sum of multiple terms with varying powers for the variables
- **Systems of Linear Equations:** Collection of two or more linear equations
- **Exponential Functions:** Function of the form $f(x) = a \cdot b^x$
- **Logarithmic Functions:** Function of the form $log_b(x) = y$
- **Inverse Functions:** A function that reverses the other function
- **Composite Functions:** An operation that takes two functions and creates a new function
- **Rational Exponents:** Expressions with exponents that are a rational number
- **Inequalities:** Equation where both sides are not equal
- **Graphs:** A diagram that illustrates the relationship between multiple variables across axes

# Quadratic Equations

**Definition**: Equations that can be rearranged into standard form

Standard Form: $ax^2 + bx + c = 0$

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Quadratic Factor: $a(x - r)(x - s) = 0$

Steps to solve Quadratic Formula:

1. Get quadratic equation into standard form
2. Substitute coefficients for a, b, and c in quadratic formula
3. Solve for x
4. Substitute the results into the quadratic factor equation for r and s

*Note: r and s in the quadratic factor equation are from solving the quadratic formula

# Quadratic Equations

Test yourself with these example problems.

**Example 1** – Factor $x^2 + 7x + 10 = 0$

**Example 2** – Factor $2x^2 - 27x + 56 = 0$

**Example 3** – Factor $2x^2 + 3x + 10 = 0$

# Quadratic Equations

## Example 1: Factor $x^2 + 7x + 10 = 0$

Solution:

Step 1 – Quadratic formula

$$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(1)(10)}}{2(1)}, where\ a = 1, b = 7, c = 10$$

Step 2 – Solve for x

$$x = \frac{-7 \pm \sqrt{49 - 40}}{2} = \frac{-7 \pm 3}{2}$$

$$x = \frac{-7 - 3}{2}, \frac{-7 + 3}{2}$$

$$x = -5, -2$$

Step 3 – Quadratic Factorization

$$(x - (-5))(x - (-2)) = 0, where\ r = -5\ and\ s = -2$$

$$(x + 5)(x + 2) = 0$$

# Quadratic Equations

## Example 2: Factor $2x^2 - 27x + 56 = 0$

Solution:

Step 1 – Quadratic formula

$$x = \frac{-(-27) \pm \sqrt{(-27)^2 - 4(2)(56)}}{2(2)}$$

Step 2 – Solve for x

$$x = \frac{27 \pm \sqrt{729 - 448}}{4} = \frac{27 \pm \sqrt{281}}{4}$$

$$x = \frac{27 - \sqrt{281}}{4}, \frac{27 + \sqrt{281}}{4}$$

Step 3 – Quadratic Factorization

$$\left(x - \left(\frac{27 - \sqrt{281}}{4}\right)\right)\left(x - \left(\frac{27 + \sqrt{281}}{4}\right)\right) = 0$$

$$\left(x + \frac{-27 + \sqrt{281}}{4}\right)\left(x + \frac{-27 - \sqrt{281}}{4}\right) = 0$$

# Quadratic Equations

## Example 3: Factor $2x^2 + 3x + 10 = 0$

Solution:

Step 1 – Quadratic formula

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(10)}}{2(2)}$$

Step 2 – Solve for x

$$x = \frac{-3 \pm \sqrt{9 - 40}}{4} = \frac{-3 \pm 31i}{4}$$

$$x = \frac{-3 - 31i}{4}, \frac{-3 + 31i}{4}$$

Step 3 – Quadratic Factorization

$$\left(x - \left(\frac{-3 - 31i}{4}\right)\right)\left(x - \left(\frac{-3 + 31i}{4}\right)\right) = 0$$

$$\left(x + \frac{3 + 31i}{4}\right)\left(x + \frac{3 - 31i}{4}\right) = 0$$

# Determinants

**Definition**: A scalar value of a function of a square matrix

2x2 Matrix Determinant formula:

$$|A| = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

3x3 Matrix Determinant formula:

$$|A| = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a\begin{bmatrix} e & f \\ h & i \end{bmatrix} - b\begin{bmatrix} d & f \\ g & i \end{bmatrix} + c\begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

$$|A| = a(ei - fh) - b(di - fg) + c(dh - eg)$$

$$|A| = aei - afh - bdi + bfg + cdh - ceg$$

# Determinants

## Test yourself with these example problems.

Example: Find the determinant of the formula below

- 2x2 Matrix Determinant formula:

$$|A| = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

- 3x3 Matrix Determinant formula:

$$|A| = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a\begin{bmatrix} e & f \\ h & i \end{bmatrix} - b\begin{bmatrix} d & f \\ g & i \end{bmatrix} + c\begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

$$|A| = a(ei - fh) - b(di - fg) + c(dh - eg)$$

$$|A| = aei - afh - bdi + bfg + cdh - ceg$$

# Linear Equations

**Definition**: Equation with two variables that creates a straight line on a graph

Example with one variable: $ax + b = 0$

Example with two variables: $ax + by + c = 0$

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

# Conjugates

**Definition**: Two pairs of binomials with the same terms where the sign between the two terms is the opposite

Expression $a-b$, conjugate $a+b$

Steps to find the conjugate

1. Change sign

# Conjugates

Test yourself with these example problems.

Example: Find the conjugate of $x^2 + 5$

Multiplying conjugates can create squares where $(a + b)(a - b) = a^2 - b^2$

Example: Factor the expression $x^2 - 9$

# Conjugates

Test yourself with these example problems.

Example: Find the conjugate of $x^2 + 5$

- Solution: $x^2 - 5$

Multiplying conjugates can create squares where $(a + b)(a - b) = a^2 - b^2$

Example: Factor the expression $x^2 - 9$

- Solution: $(x + 3)(x - 3)$

# Polynomials

**Definition**: An expression of the sum of multiple terms with varying powers for the variables

Example: $x^3 - 2x^2 + 7x + 9$

# Systems of Linear Equations

**Definition**: Collection of two or more linear equations

The solution to the system are the values of the variables that satisfy all equations.

Methods to solve system: substitution and elimination

Systems of linear equations either have one solution, no solutions, or infinitely many solutions.

# Systems of Linear Equations

**Definition**: Collection of two or more linear equations

Example system of linear equations

$$2x + 3y - z = 13$$
$$-x - 3y + 7z = -29$$
$$5x + 9y = 0$$

# Systems of Linear Equations

Test yourself with this example problem.

$3x-y=7$
$2x+3y=1$

Solving for y in the first equation gives us

$Y=3x-7$

Substituting y into the second equation gives us

$2x+3(3x-7)=1$
$2x+9x-21=1$
$11x=22$
$X=2$

Plugging this value of x into the first equation gives us

$3(2)-y=7$
$6-y=7$
$-y=1$
$Y=-1$

So our solution is $x=2$, $y=-1$

# Systems of Linear Equations

Test yourself with this example problem.

$x-y=6$
$-2x+2y=1$

Solving for x in the first equation gives us

$x=6+y$

Plugging this into the second equation gives us

$-2(6+y)+2y=1$
$-12-2y+2y=1$
$-12=1$

$-12 \neq 1$, so there are no solutions to this set of equations.

# Systems of Linear Equations

Test yourself with this example problem.

x+5y=-1
-5x-25y=5

Multiplying the first equation by 5 gives us
5x+25y=-5

Adding the two equations yields
5x+25y=-5
-5x-25y=5
0=0

Since 0=0, there are infinitely many solutions to this system.

To find one, plug in any value of x into either equation to give you your corresponding y value

# Exponential Functions

**Definition:** Function of the form $f(x) = a \cdot b^x$

Common Examples:

$$f(x) = 2^x$$

$$f(x) = \left(\frac{1}{2}\right)^x = \frac{1}{2^x} = 2^{-x}$$

$$f(x) = e^x$$

# Logarithmic Functions

**Definition:** Function of the form $log_b(x) = y$

Natural Logarithm: $f(x) = log_e(x) = \ln(x)$

Change of Base formula: $log_a x = \frac{log_b x}{log_b a}$

# Logarithm Properties

Product Property:

- $log_a(x \cdot y) = log_a(x) + log_a(y)$
- $ln\,(x \cdot y) = \ln(x) + \ln(y)$

Quotient Property:

- $log_a\left(\frac{x}{y}\right) = log_a(x) - log_a(y)$
- $ln\left(\frac{x}{y}\right) = ln(x) - ln(y)$

Power Property:

- $log_a(x^n) = n \cdot log_a(x)$
- $ln(x^n) = n \cdot \ln(x)$

# Inverse Functions

**Definition**: A function that reverses the other function

Example:

- The inverse function of $f(x) = 2x + 3$ is $f^{-1}(y) = \frac{y-3}{2}$

# Inverse Functions

Test yourself with these example problems.

y=2x+3

Switching the positions of x and y gives us

x=2y+3

Subtracting 3 to both sides gives us

x-3=2y

Dividing both sides by 2 gives us the function

x/2-3/2=y

y=x/2-3/2

# Composite Functions

Test yourself with these example problems.

Find $(g \circ f)(x) = g(f(x))$ for the two functions $f(x) = x + 5$ and $g(x) = x^3$

$$(g \circ f)(x) = g\big(f(x)\big) = (x + 5)^3$$

Find $(f \circ g)(x) = f(g(x))$ for the two functions $f(x) = x + 5$ and $g(x) = x^3$

$$(f \circ g)(x) = f\big(g(x)\big) = x^3 + 5$$

# Rational Exponents

**Definition**: Expressions with exponents that are a rational number

Examples in exponent form: $8^{\frac{1}{2}}, 153^{\frac{1}{3}}, 16x^{\frac{1}{4}}$

Expressions with a rational exponent can be represented in radical form

Examples in radical form: $\sqrt{8}, \sqrt[3]{153}, \sqrt[4]{16x}$

# Inequalities

**Definition**: Equation where both sides are not equal

Symbols:

- $<$ Less Than
- $>$ Greater Than
- $\neq$ Not Equal
- $\leq$ Less Than or Equal To
- $\geq$ Greater Than or Equal To

Example:

$-2x - 3 < 10x^2 + 3$ when $x$ is a positive integer

# Graphs

**Definition**: A diagram that illustrates the relationship between multiple variables across axes

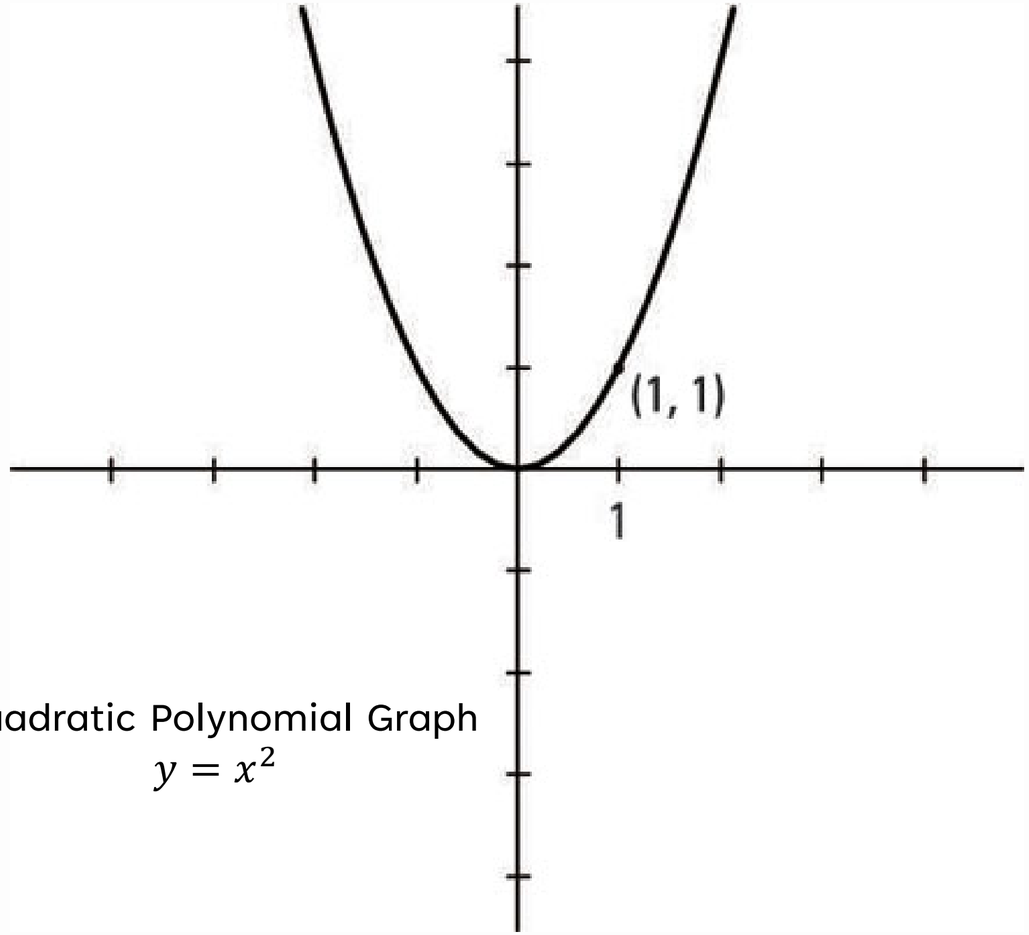
Different types of equations and functions can be represented and illustrated on a graph

Common graphs are:

- Quadratic polynomial
- Cubic polynomial
- Line
- Absolute value
- Reciprocal
- Square root
- Cube root
- Exponential function
- Logarithmic function

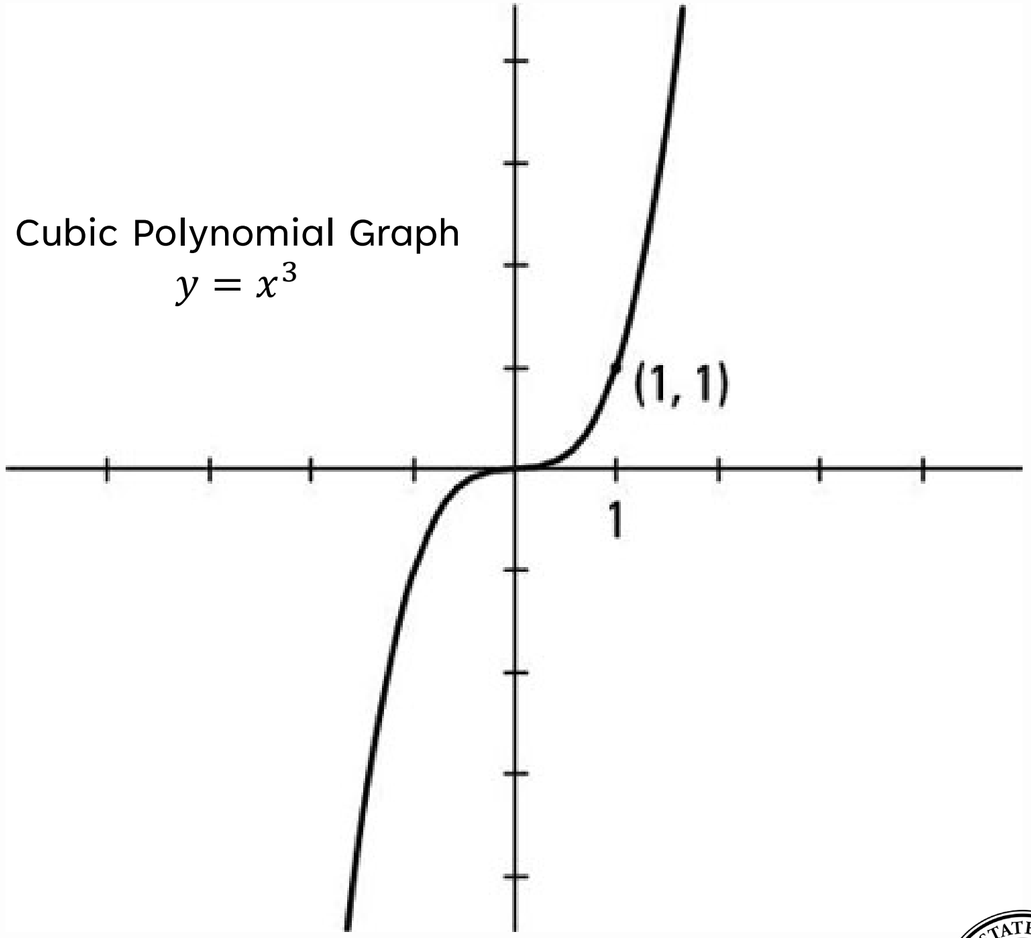
# Graphs

**Examples**: Quadratic Polynomial & Cubic Polynomial



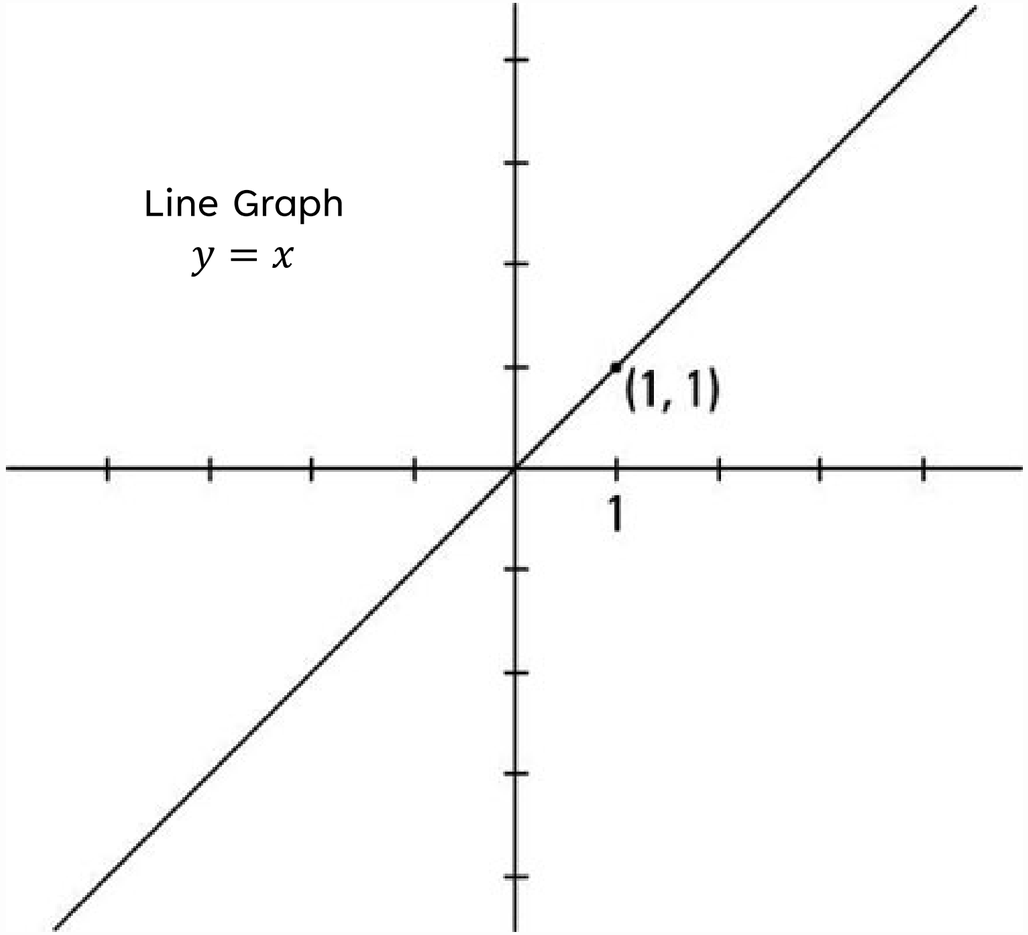

Quadratic Polynomial Graph
$y = x^2$

Cubic Polynomial Graph
$y = x^3$

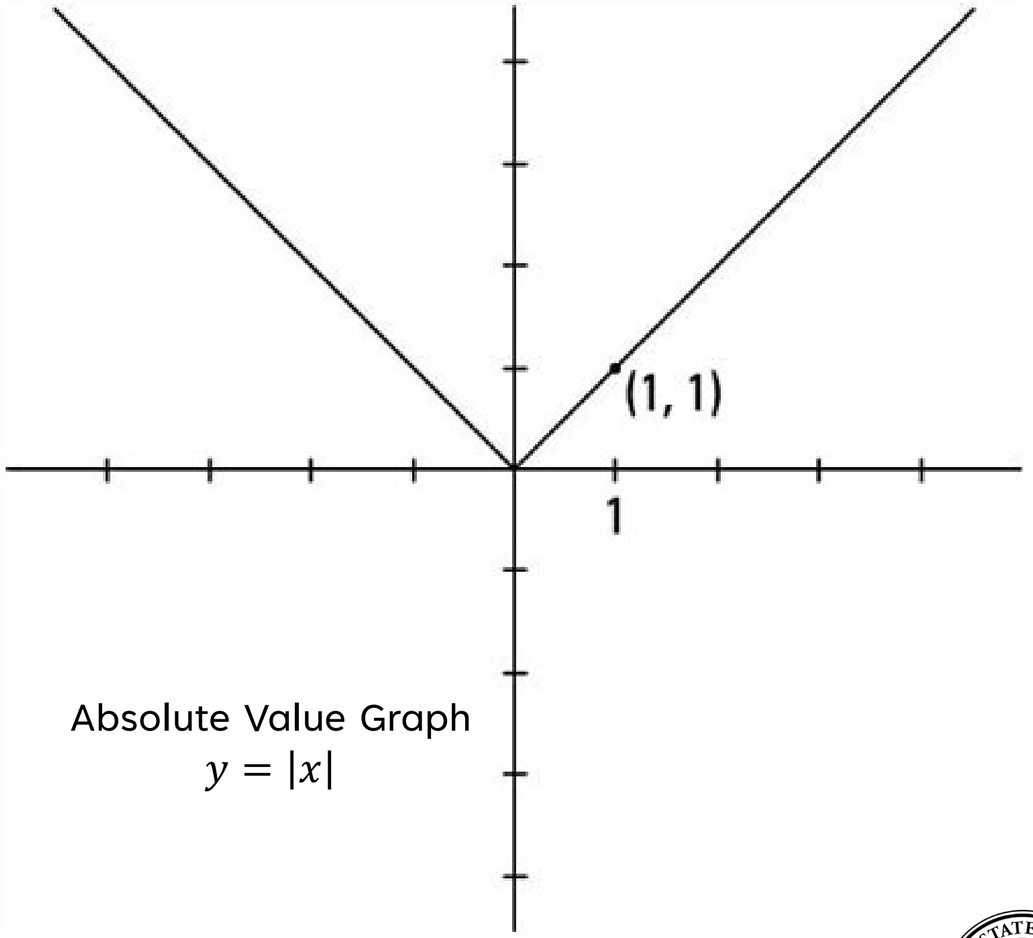


# Graphs

**Examples**: Line & Absolute Value




Line Graph
$y = x$



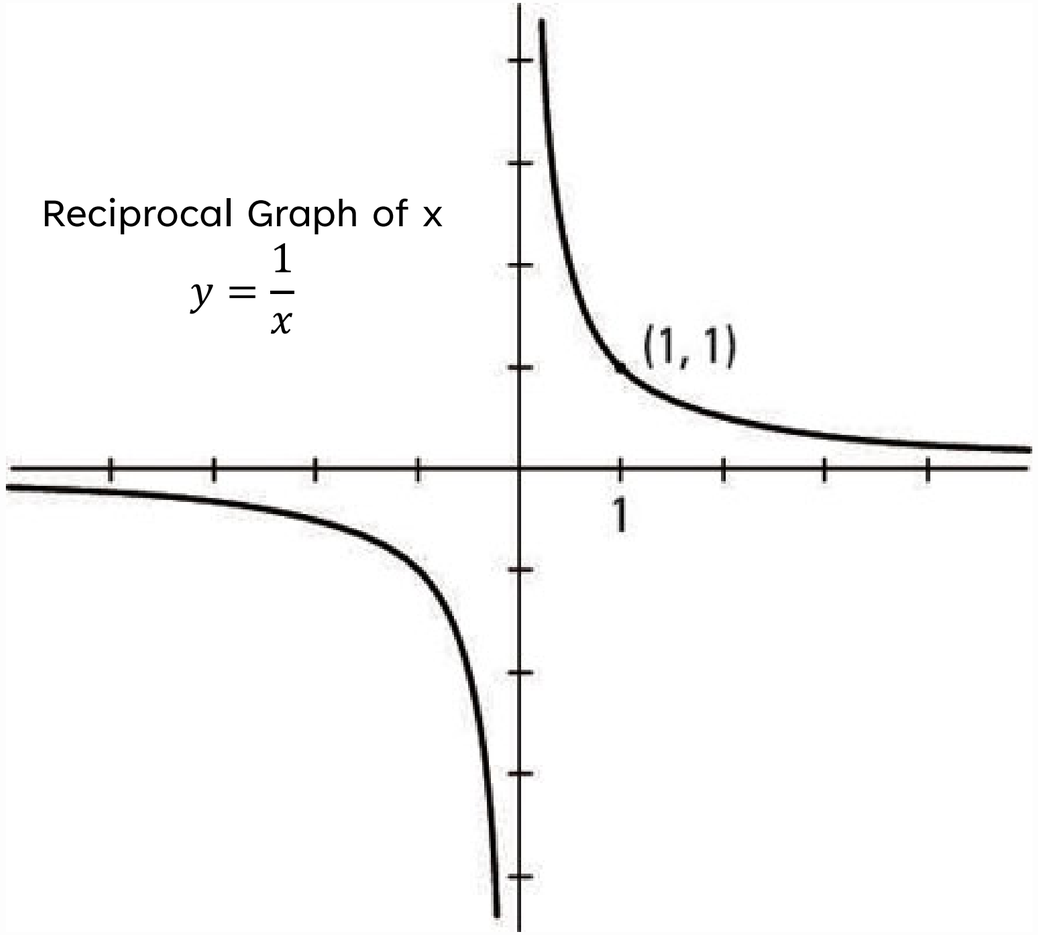

Absolute Value Graph
$y = |x|$

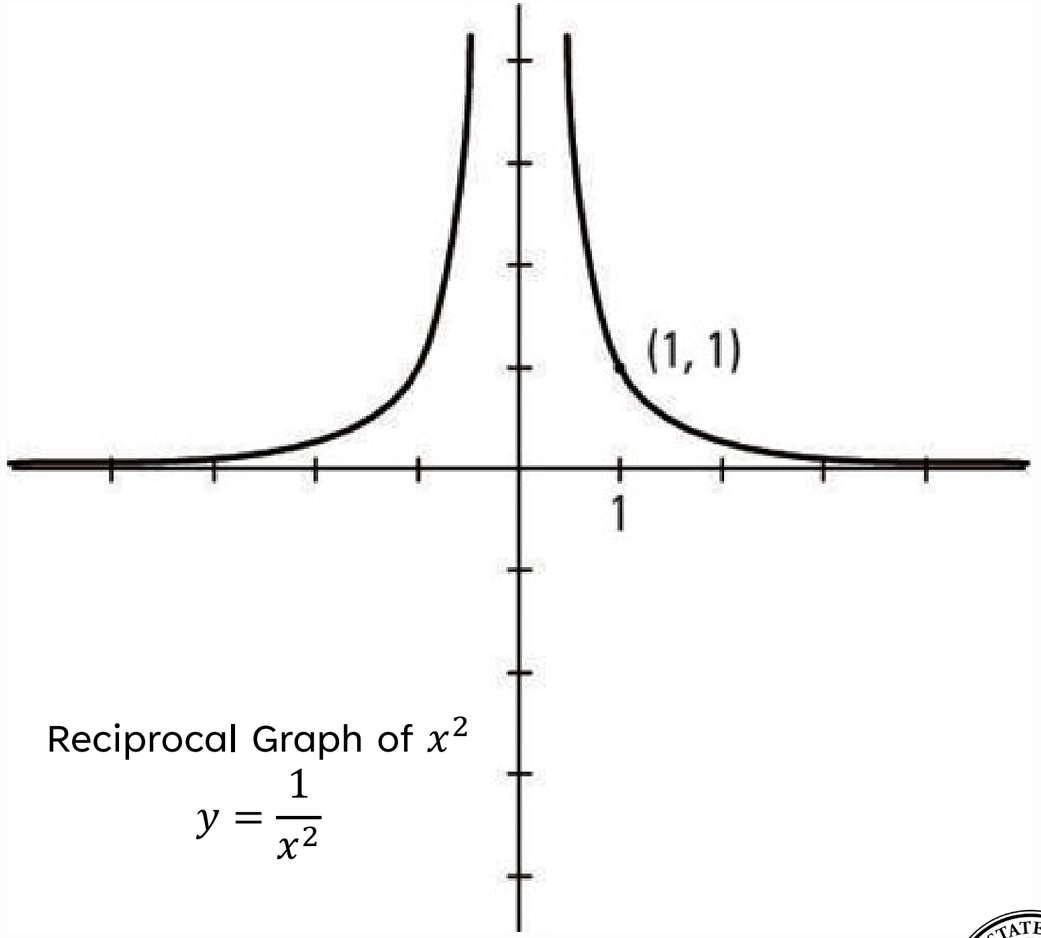
# Graphs

## **Examples**: Reciprocal Graphs

Reciprocal Graph of x

$$y = \frac{1}{x}$$




Reciprocal Graph of $x^2$

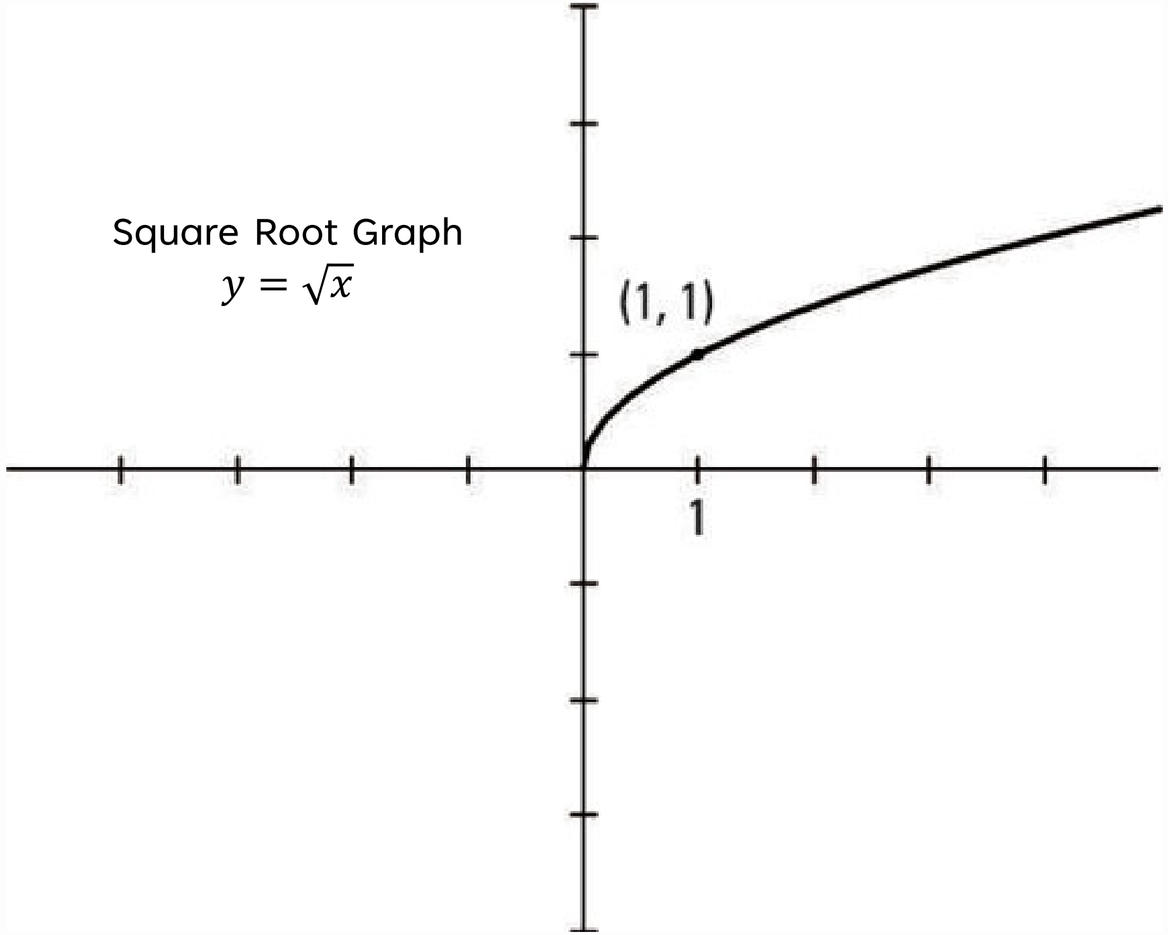
$$y = \frac{1}{x^2}$$

# Graphs

**Examples:** Square Root & Cube Root Graphs

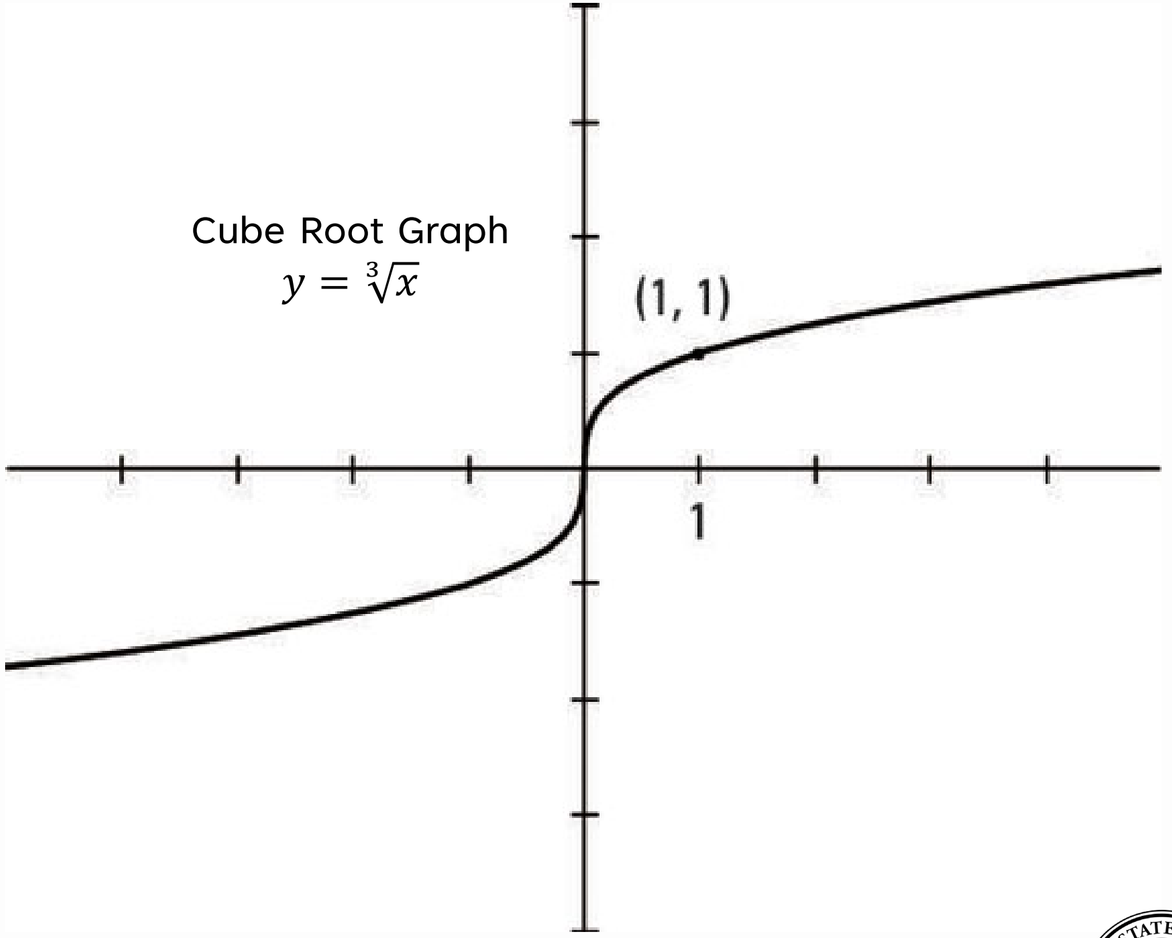
Square Root Graph

$y = \sqrt{x}$



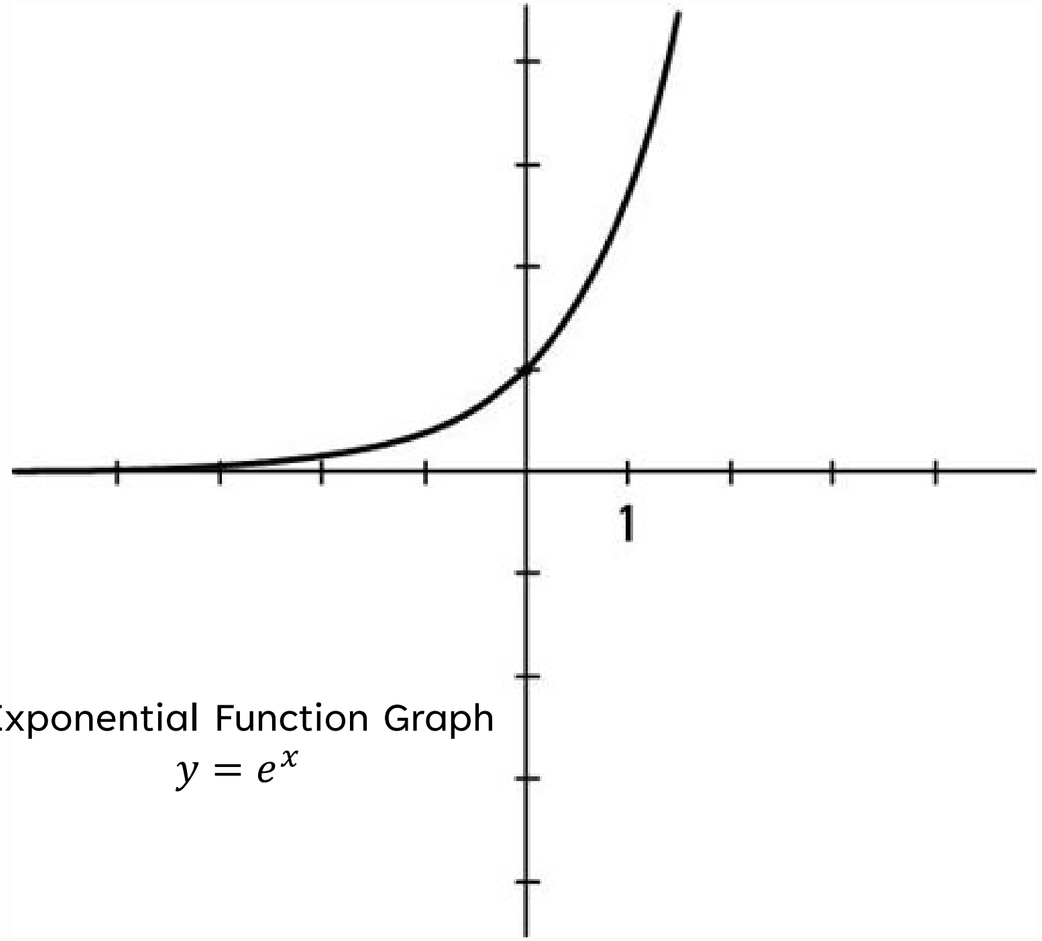

Cube Root Graph

$y = \sqrt[3]{x}$



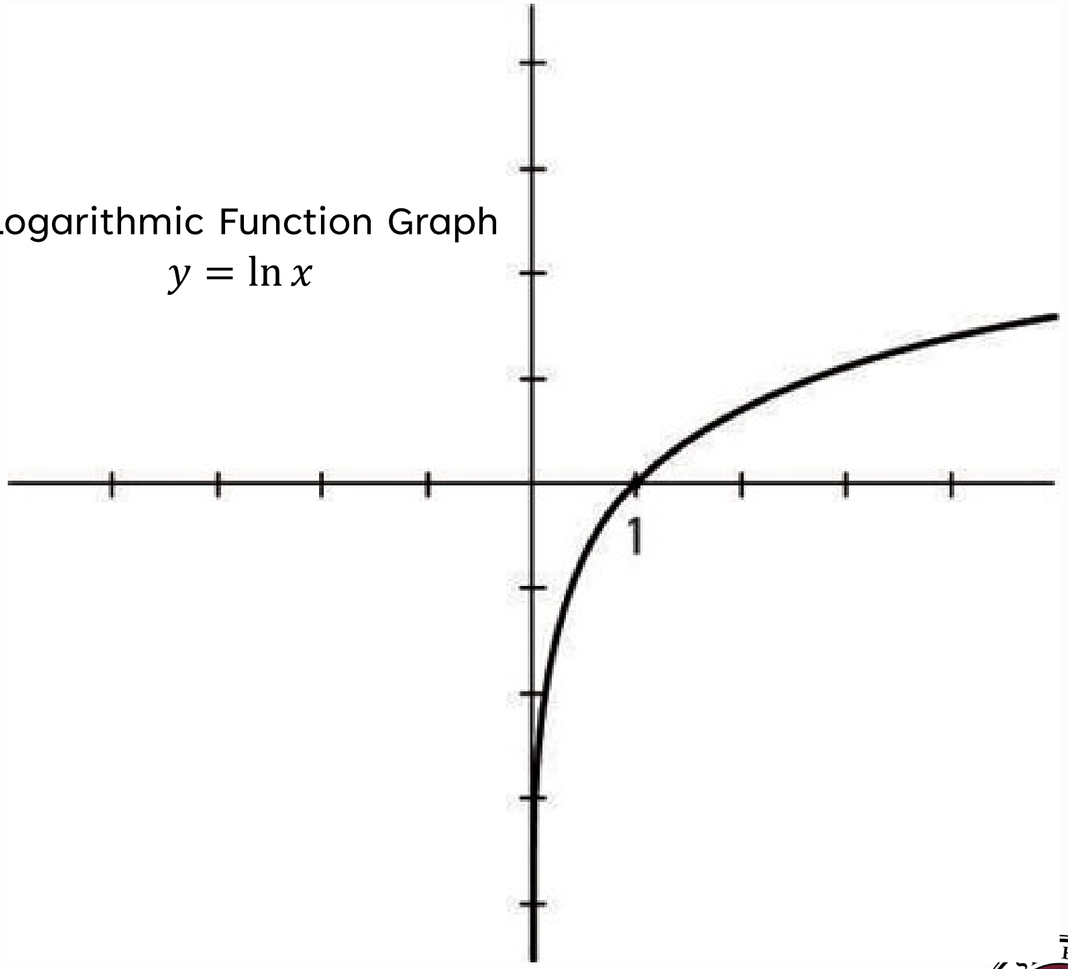
# Graphs

**Examples**: Exponential Function & Logarithmic Function




Exponential Function Graph
$y = e^x$

Logarithmic Function Graph
$y = \ln x$



# References & Resources

Use these links for more information or come visit one of the FSU tutoring programs for one-on-one help!

- **Paul's Online Math Notes** (tutorial.math.lamar.edu/): The intent of this site is to provide a complete set of free online (and downloadable) notes and/or tutorials for math classes. They're written the notes/tutorials to be accessible to anyone wanting to learn the subject.

- **Desmos** (desmos.com/calculator): To create a new graph, just type your expression in the expression list bar. As you are typing your expression, the calculator will immediately draw your graph on the graph paper.

- **CalcPlot3D** (c3d.libretexts.org/CalcPlot3D/index.html): This dynamic Java applet allows the user to simultaneously plot multiple 3D surfaces, space curves, parametric surfaces, vector fields, contour plots, and more in a freely rotatable graph.

# Tell us how we're doing!

The QR code below will take you to a survey about the Readiness Review Series.

We're always interested in improving, so we're asking for your feedback.

What was your experience like? Is there anything we should add, change, or remove?

Do you have ideas for how this program should be expanded in the future?

Scan the code and fill out the short survey to let us know!