Readiness Review Series

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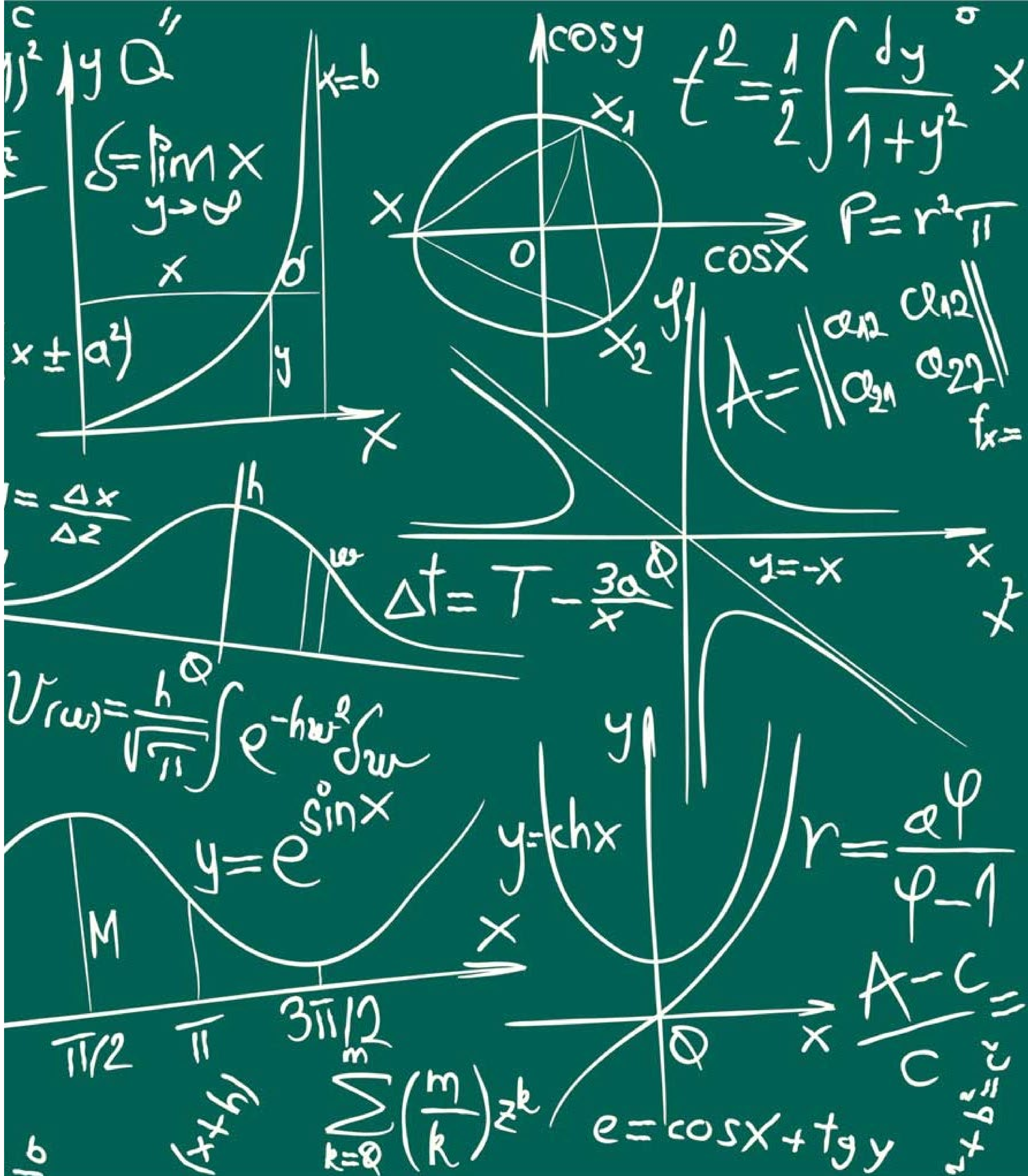


Tips for Math Success

Believe that your intelligence can grow.

The first battle in learning anything is believing you're capable. Cultivating a growth mindset (i.e. believing you're fully capable of mastering new material) will help immeasurably in achieving academic success.

Practice by teaching someone else a difficult concept.

Explaining a concept to someone else will help you work through the material yourself and reinforce the information in your own mind. Once you can successfully teach someone else the concept, you know you've mastered it.

Get tutoring weekly.

ACE, CARE, and Libraries' Learning District all offer free help for math courses. Visit tutoring.fsu.edu for more information on what you can get help with and how to access those programs.

Memorize key formulas/theorums.

Having crucial formulas and concepts on hand will help you navigate the more complex concepts to come.

Tips for Math Success

Always attend class.

This is the best chance to hear extra explanations, ask questions, and gain a stronger understanding of how each concept fits into the overall subject matter.

Start homework the day it is assigned.

The best practice is to complete homework problems without using example problems as a guide or copying answers from another source. Also, even when it is not worth points, you should focus on mastering the content of these assignments.

Make your schedule work for you.

If you are taking 15 credits this semester, create a weekly study schedule with 25 hours of study time during the week. It's also better to create 30-minute time blocks per class throughout the week as opposed to cramming. You'll remember a lot more when exposed to the material multiple times by practicing problems 2 or 3 times outside of class each week.

Be an active reader.

When you read your textbook, paraphrase each paragraph or section to ensure you understand. It can also help to color code your notes to help you identify what you do not understand and give you the chance to ask for clarification later.

Tips for Math Success

Ask for help.

Spend a little bit of time trying to resolve it yourself, but don't spin your wheels. If something doesn't make sense or you feel stuck on a problem or concept, reach out to the instructor or the TAs for guidance. Visiting your instructors regularly in office hours will help you to develop better communication channels and to master the content you do not understand.

If you work with a tutor, make sure you have done some legwork before the tutoring session.

Make sure you know where you could use the additional help so that your tutoring session is effective and efficient.

Work well in advance of deadlines.

Last-minute emergencies and conflicts can never be predicted. You don't want to miss out on earning points because of procrastination.

Be comfortable being uncomfortable.

Learning takes time, and until we have mastered something, we may often lack confidence in our abilities and our knowledge. The more time you spend studying something, the more comfortable you will become with the topic. However, be patient with yourself as you are learning.

MAC1140 – Precalculus

Table of Contents

Vocabulary Review

Begin by reviewing these key terms.

- Trigonometry
  "Triangle Measurement"

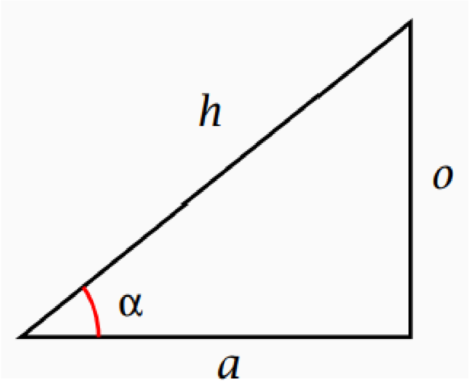
When one first meets the trigonometric functions, they are presented in the context of ratios of sides of a right-angled triangle, where $a^2 + o^2 = h^2$:

We have

$$\sin\alpha = \frac{o}{h} = \frac{\text{opposite}}{\text{hypotenuse}},$$

$$\cos\alpha = \frac{a}{h} = \frac{\text{adjacent}}{\text{hypotenuse}}, \text{ and}$$

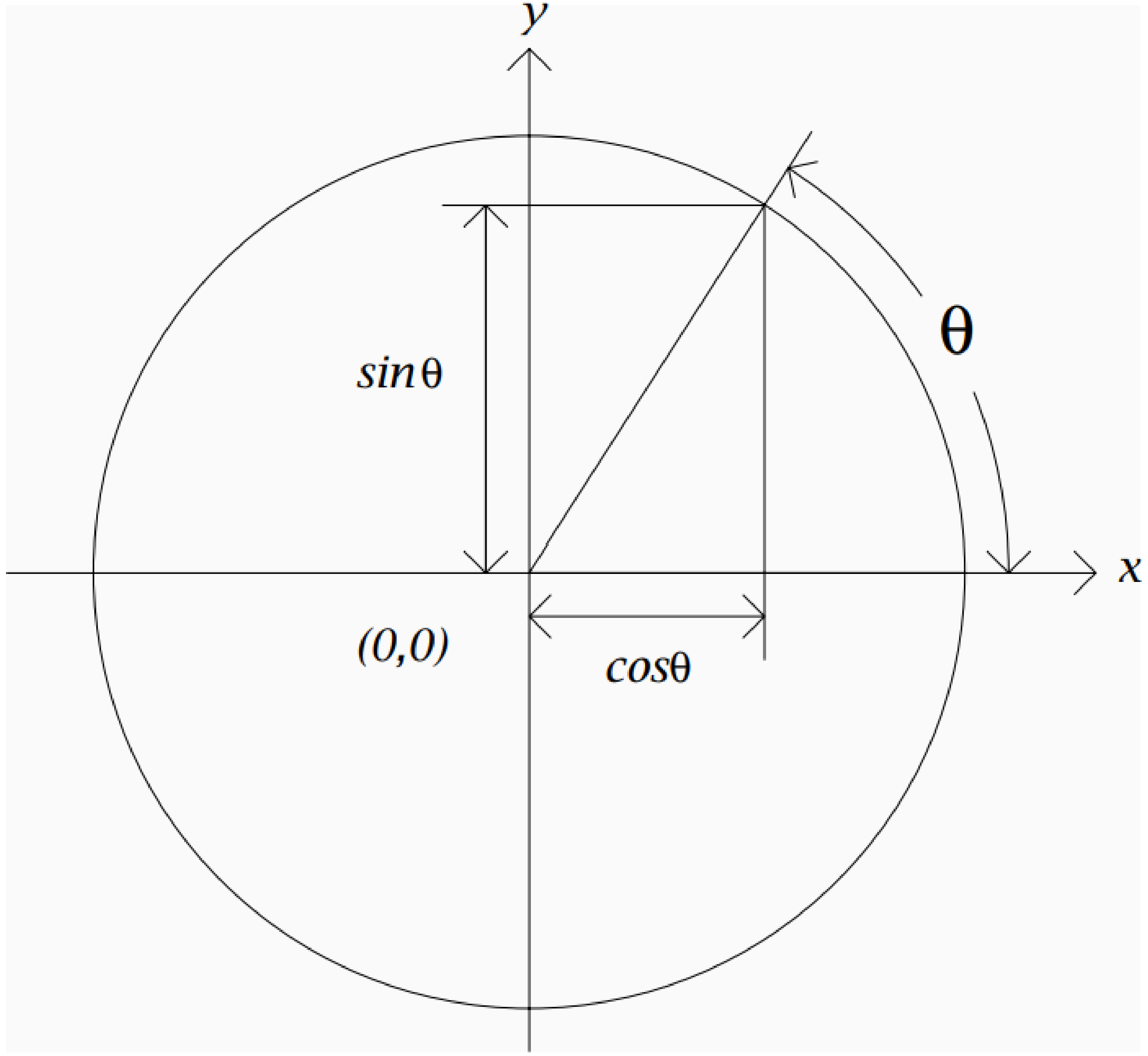
$$\tan\alpha = \frac{o}{a} = \frac{\text{opposite}}{\text{adjacent}},$$

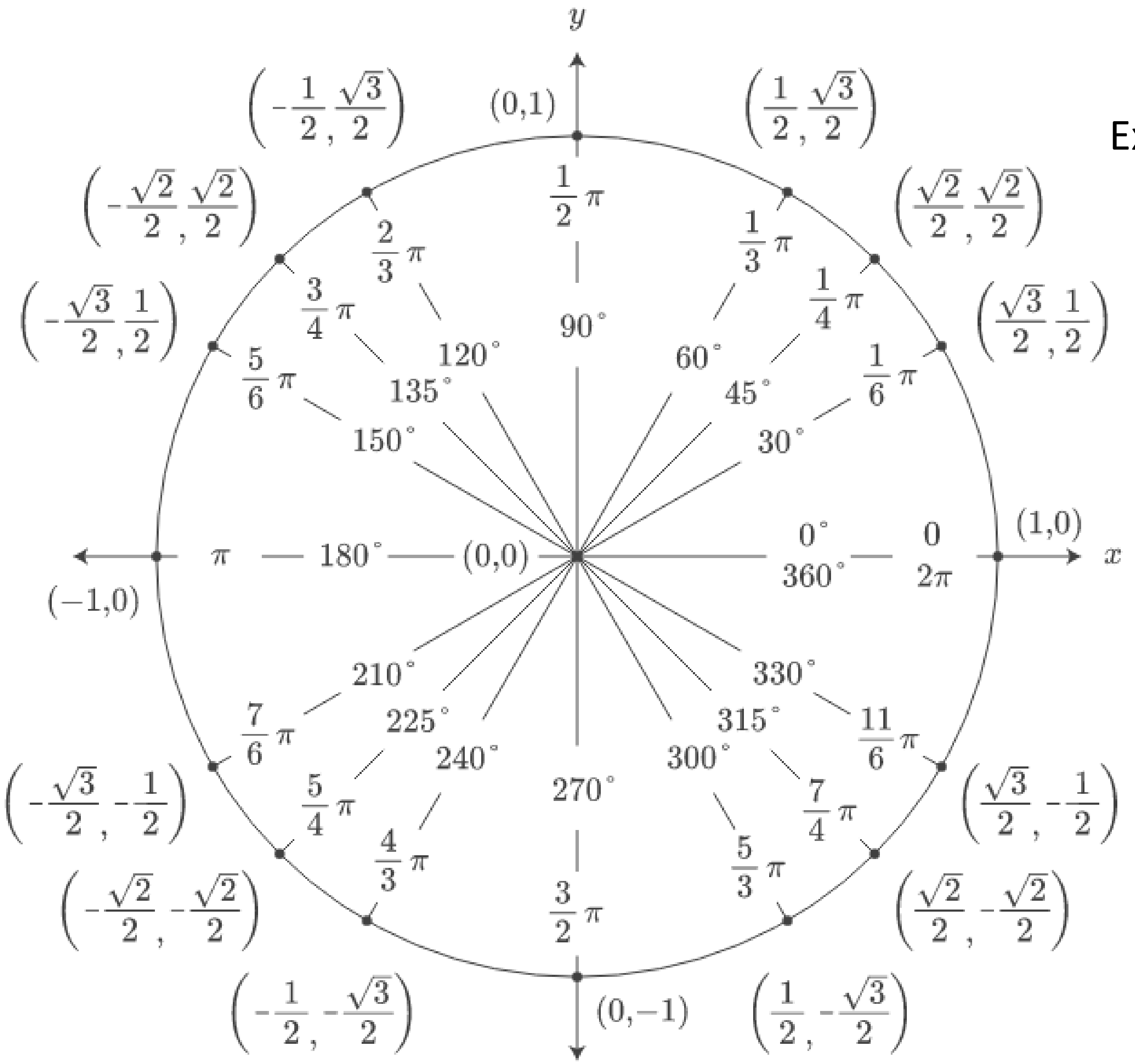


- Unit Circle
  The basis of all trigonometric functions, based off the equation for circle/sphere: $x^2 + y^2 = 1$
- Radian
  Angle subtended by a circular arc whose length equals the radius of the circle. The radian is determined by the amount of circular arc around the "circle", or how big the angle made starting at the x-axis is.

Unit Circle

It's a sad truth, but you **need** to memorize the first quadrant values, at the very least.





$\cos\theta = x$

$\sin\theta = y$

Ex: $\cos\frac{\pi}{3} = \frac{1}{2}$

Imaginary Unit

Definition: One property of a real number is that its square is nonnegative

For example, there is no real number x such that $x^2 = -1$. To remedy this situation, we introduce a new number called an *imaginary unit*.

- The **imaginary unit**, $\boldsymbol{i}$, is defined to be a solution for the equation $\boldsymbol{x^2 = -1}$. We write $\boldsymbol{i} = \sqrt{-\mathbf{1}}$.

- The solution of the equation $\boldsymbol{x^2} = -\mathbf{1}$ is $\{i, -i\}$, and i is the **principal square root** of $-\mathbf{1}$.

Imaginary Unit

Test yourself with these examples.

Example 1: Simplify $\sqrt{-12}$.

Example 2: Simplify i^{27}.

Example 3: Simplify i^{-27}.

Imaginary Unit

Test yourself with these examples.

Example 1: Simplify $\sqrt{-12}$.

Is this an accurate solution? $\sqrt{-12} = \sqrt{12} \cdot \sqrt{-1} = 2\sqrt{3}i$

Example 2: Simplify i^{27}.

Is this an accurate solution? $i^{27} = i \cdot \left(i^2\right)^{13} = -i$

Example 3: Simplify i^{-27}.

Is this an accurate solution? $i^{-27} = i \cdot \left(i^{-2}\right)^{13} = -i$

Complex Numbers

Definition: a number that may be written in the form $\boldsymbol{a+bi}$.

This form is called the standard form of a complex number. The real number $\boldsymbol{a}$ is called the real part and $\boldsymbol{b}$ is called the imaginary part.

For example, for the complex number $\mathbf{4+3}\boldsymbol{i}$, $\mathbf{4}$ is the real part and $\mathbf{3}$ is the imaginary part.

Complex Conjugate

Definition: a number that may be written in the form $\boldsymbol{a+bi}$.

The **complex conjugate** of $\mathbf{z} = \boldsymbol{a} + \boldsymbol{bi}$ is $\bar{\mathbf{z}} = \boldsymbol{a} - \boldsymbol{bi}$.

Notice that the product of conjugates is a real number. In particular,

$$\boldsymbol{z\bar{z}} = (\mathbf{a} + \mathbf{bi})(\mathbf{a} - \mathbf{bi}) = \boldsymbol{a}^2 + \boldsymbol{b}^2.$$

For example, if $\mathbf{z} = \mathbf{2} + \mathbf{3}\boldsymbol{i}$ and $\bar{\mathbf{z}} = \mathbf{2} - \mathbf{3}\boldsymbol{i}$, then

$$\boldsymbol{z\bar{z}} = \mathbf{2}^2 + \mathbf{3}^2 = \mathbf{13}.$$

Algebra of Complex Numbers

Test yourself with these examples

Example 1: $(3 + 4\mathrm{i}) + (4 - 5\mathrm{i})$

Example 2: $(3 + 4\mathrm{i}) - (4 - 5\mathrm{i})$

Example 3: $(3 + 4\mathrm{i})(4 - 5\mathrm{i})$

Example 4: $\frac{3+4i}{4-5i}$

Algebra of Complex Numbers

Test yourself with these examples

Example 1: $(3 + 4\mathrm{i}) + (4 - 5\mathrm{i}) = 7 - \mathrm{i}.$

Example 2: $(3 + 4\mathrm{i}) - (4 - 5\mathrm{i}) = -1 + 9\mathrm{i}.$

Example 3: $(3 + 4\mathrm{i})(4 - 5\mathrm{i}) = 12 - 15\mathrm{i} + 16\mathrm{i} - 20i^2 = 32 + i.$

Example 4: $\frac{3+4i}{4-5i} = \frac{3+4i}{4-5i} \cdot \frac{4+5i}{4+5i} = \frac{12+15i+16i+20i^2}{16+25} = \frac{-8+31i}{41} = \frac{31}{41}i - \frac{8}{41}$

Complex Solutions to Quadratic Equations

The solutions to the equation $\boldsymbol{ax^2 + bx + c = 0}$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Example: Find all zeros (real and complex) of $\boldsymbol{f(x) = x^3 + 27}$.

Using the above equation, we have that:

$$\boldsymbol{x = \frac{3}{2} + \frac{5}{2}i, x = \frac{3}{2} - \frac{5}{2}i}.$$

Discriminant of a Quadratic Equation

The discriminant of the quadratic equation $\boldsymbol{ax^2 + bx + c = 0}$ is $\boldsymbol{b^2 - 4ac}$.

If the discriminant of a quadratic equations is …

- **zero**, there is one real solution,
- **positive**, there are two real solutions,
- **negative**, there are two complex conjugate solutions.

Discriminant of a Quadratic Equation

Test yourself with these examples

Example 1: Find the discriminant of the quadratic equation:

$$\mathbf{-3x^2 + 4x + 2 = 0}$$

Example 2: Find the discriminant of the quadratic equation:

$$\mathbf{3x^2 + x + 2 = 0}$$

Discriminant of a Quadratic Equation

Test yourself with these examples

Example 1: Find the discriminant of the quadratic equation:

$\mathbf{-3x^2 + 4x + 2 = 0}$

$4^2 - 4(-3)(2) = 40.$

Since 40 is positive, there are 2 real solutions.

Example 2: Find the discriminant of the quadratic equation:

$\mathbf{3x^2 + x + 2 = 0}$

$1^2 - 4(3)(2) = -23.$

Since -23 is negative, there are 2 complex solutions.

One-to-One Functions

A function is **one-to-one** if whenever you choose two different numbers $x_1$and x_2 in the domain of f, you have $f(x_1)$ and $f(x_2)$ are also different. In other words, each value of x corresponds to only one y and each value of y corresponds to only one x.

Theorem: A function f is one-to-one only if and only if you cannot draw a horizontal line passing through the graph of f more than once.

Inverse Functions

Let f be a one-to-one function. Then, there is a function denoted f^{-1} called the inverse of f such that the domain and ranges of f and f^{-1} are interchanged and $f(a) = b$ if and only if $f^{-1}(b) = a$.

If $f(x)$ and $g(x)$ are inverse functions, the domain of $f(x)$ is the same as the range of $g(x)$.

Inverse Function Properties

A function g is the inverse of f (and visa versa) if and only if $(fog)(x) = x$ and $(gof)(x) = x$. Reminder: $(fog)(x) = f\big(g(x)\big)$.

The domain of f and the range of f^{-1} and the domain of f^{-1} is the range of f.

(a, b) is a point on the graph of $y = f(x)$ if and only if (b, a) is a point on the graph of $y = f^{-1}(x)$.

The graph of $y = f^{-1}(x)$ is the reflection of the graph of $y = f(x)$ about the lone $y = x$.

Exponential Functions

Definition: a function of the form $f(x) = Ca^x$

Exponential functions are written as $f(x) = Ca^x$ where a is a real number with $a > 0$ and $a \neq 1$. The base a is called growth factor and C the initial value.

For an exponential function $f(x) = Ca^x$, the ratio of $f(x+1)$ and $f(x)$ is a constant. In fact, $\frac{f(x+1)}{f(x)} = a$, the base.

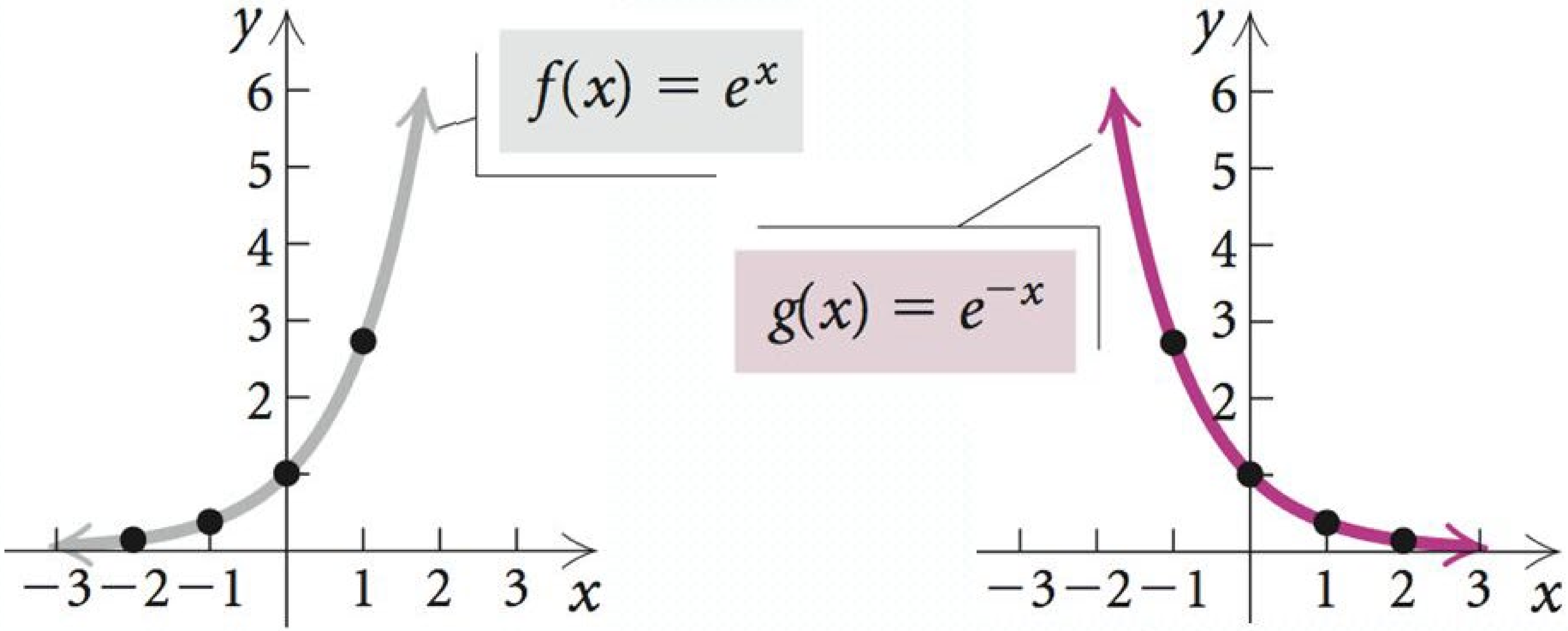
Exponential Functions

Properties of exponential functions

	$f(x) = a^x,\ a > 1$	$f(x) = a^x,\ 0 < a < 1$
Domain	$(-\infty, \infty)$	$(-\infty, \infty)$
Range	$(0, \infty)$	$(0, \infty)$
Increasing	$(-\infty, \infty)$	Never
Decreasing	Never	$(-\infty, \infty)$
y − intercept	$(0, 1)$	$(0, 1)$
Horizontal Asymptote	$y = 0$	$y = 0$

Exponential Functions

Graphs of exponential functions



The graph of g is a reflection of the graph of f across the y-axis.

Conic Sections

An introduction to the terms.

- **Conics**, an abbreviation for conic sections, are cross sections that result from the intersection of a right circular cone and a plane

- **Circles** are when the plane is perpendicular to the axis of the cone when it intersects.

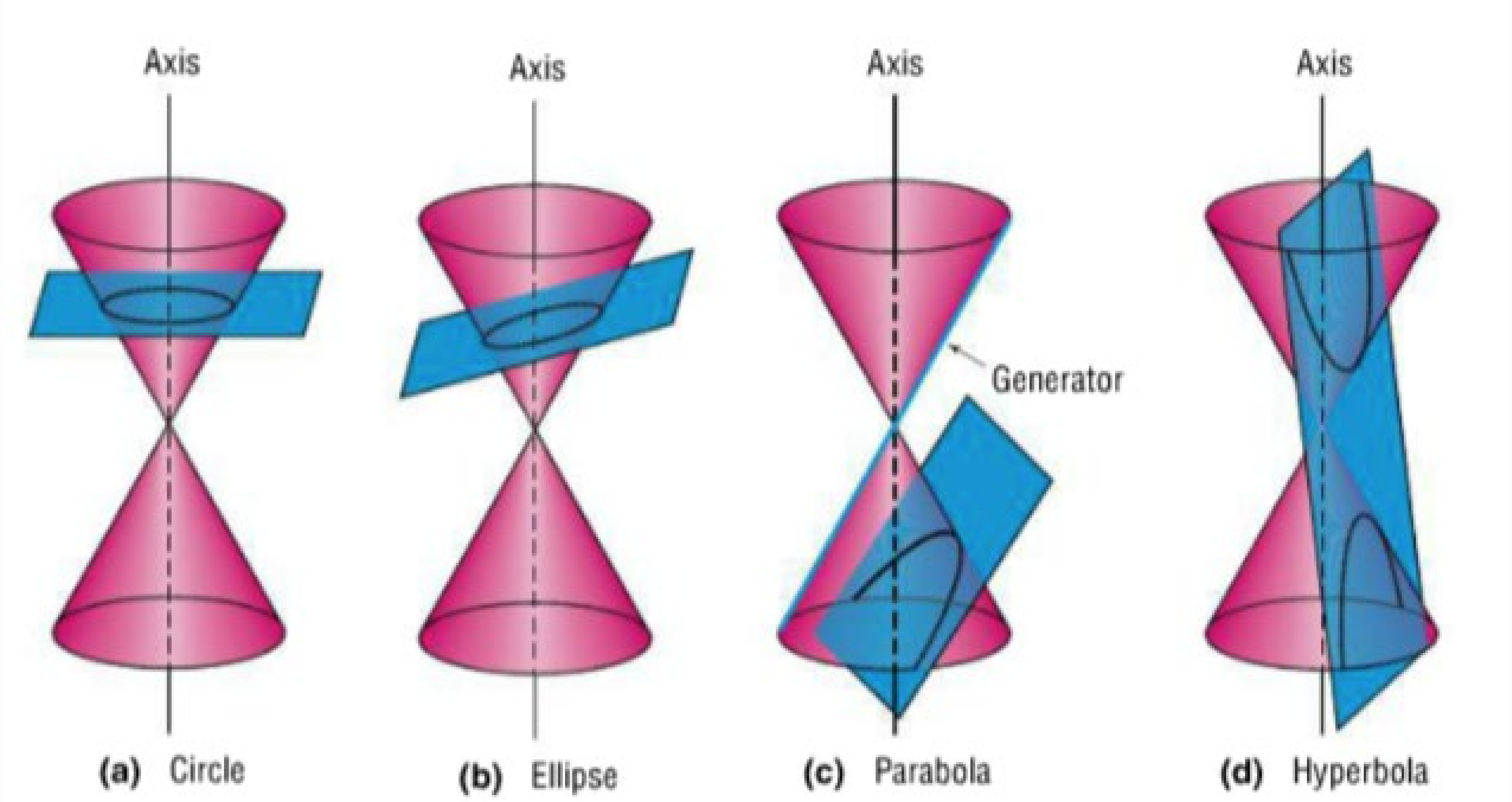
- **Ellipses** are when the plane is tilted slightly when it intersects the cone.

- **Parabolas** are when the place is tilted farther so that it is parallel to one generator

- **Hyperbolas** are when the plane intersects both parts (called nappes) of the cone

Conic Sections

An introduction to the visual representations.



Conic Sections

Applications of conic sections

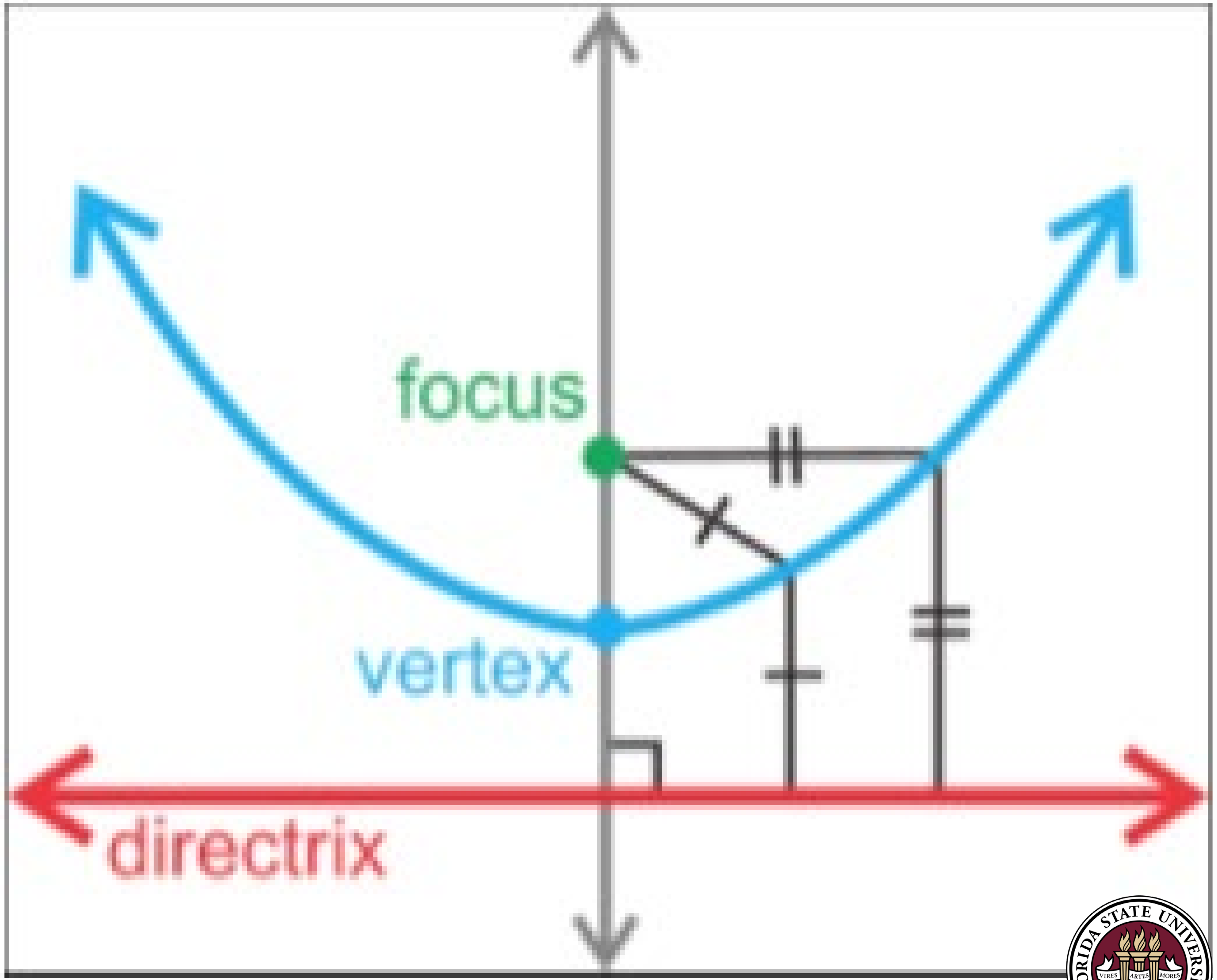
- **Parabola**: used for searchlight, satellite dishes, and telescopes
- **Ellipses**: model orbits of planets and whispering chambers
- **Hyperbolas**: used to locate lightning strikes and design nuclear cooling towers.

Parabola

Important details

A parabola is the set of all points, $P = (x, y)$ that are equidistant from a fixed point called the focus and a fixed line called the directrix.

- The focus and the directrix are equidistant from the vertex.
- The vertex and focus lie on the axis of symmetry
- The directrix and the axis of symmetry are perpendicular to each other
- The latus rectum is a line segment that is parallel to the directrix with endpoints and the parabola and the midpoint the focus
- The parabola opens away from the directrix and around the focus.



Parabolas

Parabolas with vertex at (h,k)

If a parabola is translated h units horizontally and k units vertically, the vertex will be (h, k). This translation results in the standard form of the equation with x replaced by $(x - k)$ and y replaced by $(y - k)$.

Horizontal Directrix, Vertex at (h, k)

$$(y - k) = \frac{1}{4p}(x - h)^2 \text{ or } y = \frac{1}{4p}(x - h)^2 + k$$

Vertical Directrix, Vertex at (h, k)

$$(x - h) = \frac{1}{4p}(y - k)^2 \text{ or } x = \frac{1}{4p}(y - k)^2 + h$$

Parabolas

Parabolas with vertex at (h,k)

	Horizontal Parabola	Vertical Parabola
Axis of Symmetry	$y = k$	$x = h$
Equation	$(y - k)^2 = 4a(x - h)$	$(x - h)^2 = 4p(y - k)$
Focus	$(h + p, k)$	$(h, k + p)$
Directrix	$x = h - p$	$y = k - p$
Endpoints of Focal Diameter	$(h + p, k \pm 2p)$	$(h \pm 2p, k + p)$

Parabolas

Parabolas with vertex at (0,0)

Example 1:

Find the equation of the parabola described below and sketch a graph.

focus at (0, 3) and directrix the line y = -3

The form of the parabola is $x^2 = 4ay$. Hence, the equation of the parabola is

$$x^2 = 4(3)y \rightarrow x^2 = 12y$$

Parabolas

Parabolas with vertex at (h,k) ≠ (0,0)

Example 1:

Find the equation of the parabola with focus (-1, -2) and directrix y=3.

The form of the parabola is $(x-h)^2 = 4a(y-k)$. Hence, the equation of the parabola is

$$(x+1)^2 = 4(2.5))(y-\frac{1}{2}) \rightarrow (x+1)^2 = 10(y-\frac{1}{2})$$

Ellipse

Important details about ellipses.

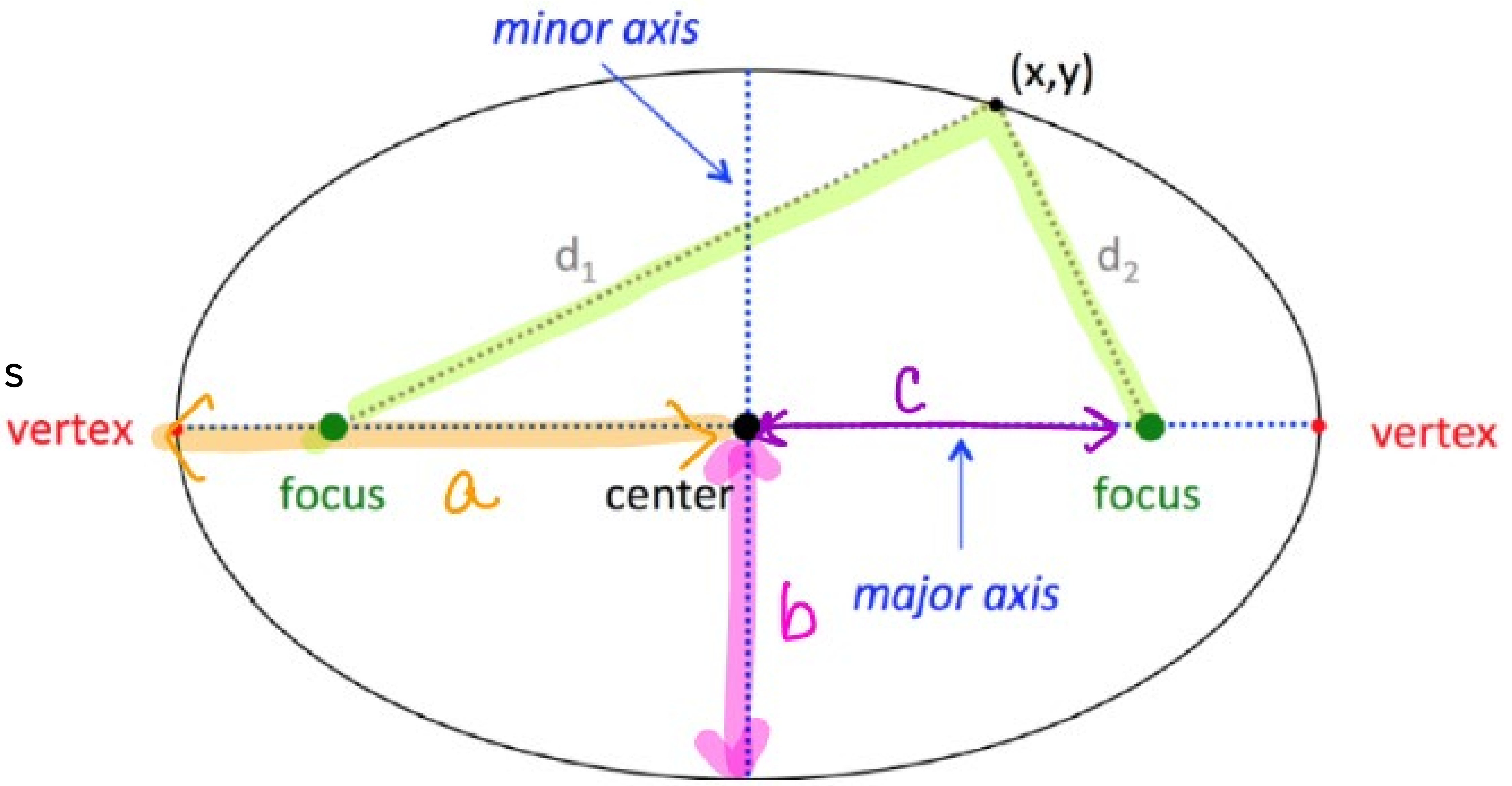
An ellipse is the set of all points, $P = (x, y)$, such that the sum of the distances between two points, called the foci of the ellipse, is constant.

- The equation of this horizontal ellipse is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

- The equation of this vertical ellipse is

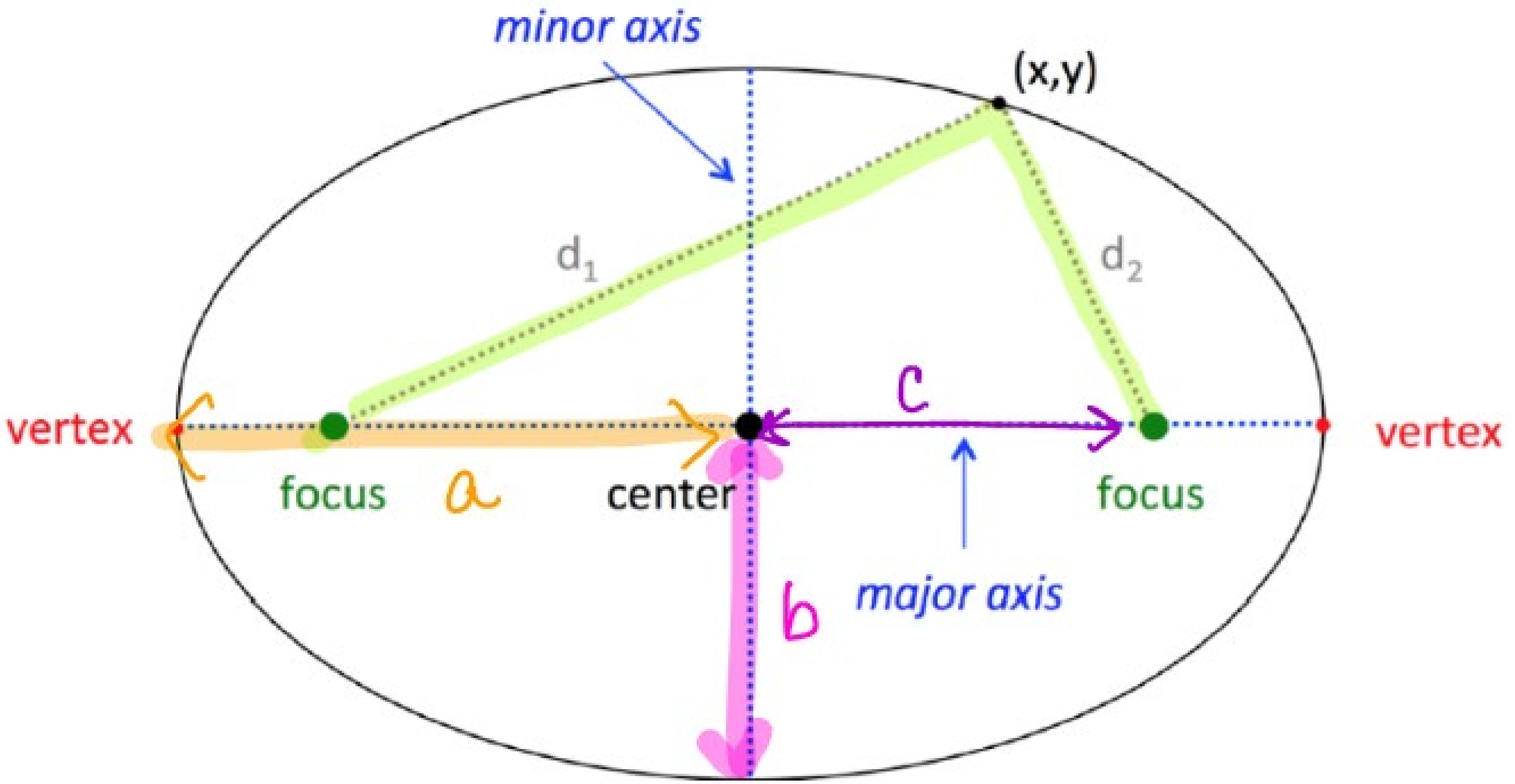
$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$



Ellipse

Important details about ellipses.

- The vertices of the ellipse are the points where the ellipse intersects the line through the foci
- The major axis is the line segment between the two vertices.
- The minor axis is the line segment perpendicular to the major axis through the center and inside the ellipse.
- The length of the major axis is 2a; the length of the minor axis is 2b.
- The major axis of an ellipse can be vertical or horizontal. It depends which variable a^2 is under the ellipse is in standard form.



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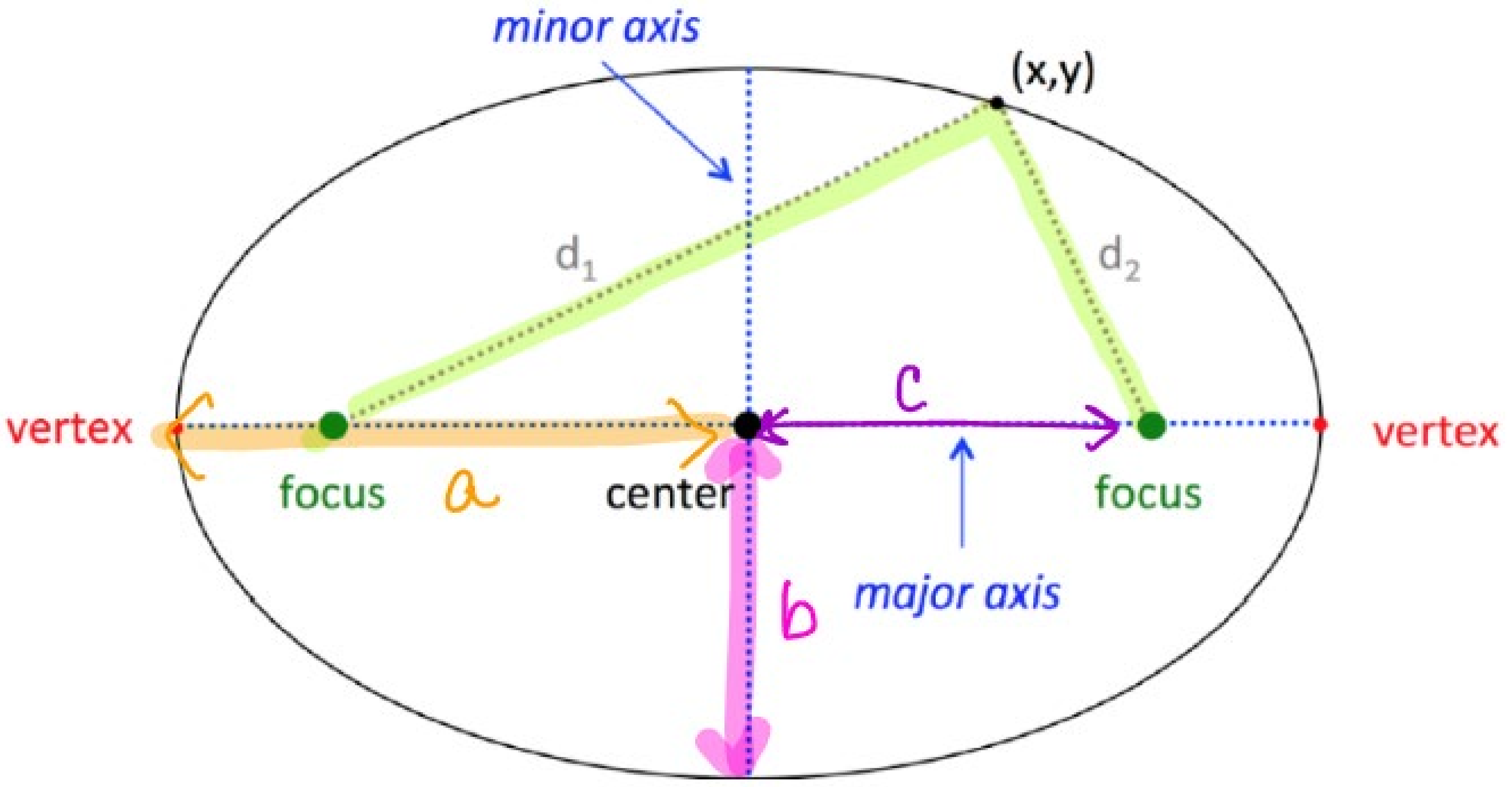

Ellipse

Standard equations for ellipses.

Equation a>b>0	Horizontal Ellipse	Vertical Ellipse
Relation between a, b and c	$a^2 = b^2 + c^2$	$a^2 = b^2 + c^2$
Major Axis	horizontal	vertical
Focus	$(h \pm c, k)$	$(h, k \pm c)$
Vertices	$(h \pm a, k)$	$(h, k \pm c)$
Intercepts and minor axis	$(h, k \pm 2p)$	$(h \pm b, k)$

Ellipse

Analyzing and graphing ellipses.

Find the center, foci, vertices, major axis, and minor axis for the ellipse:

$$\frac{(x+1)^2}{48} + \frac{(y-1)^2}{64} = 1.$$

The center of this vertical ellipse is (-1, 1), a = 8 and b = $6\sqrt{3}$ and hence c = $\sqrt{64-48} = 4$.

The foci are hence at (-1, 5) and (-1, -3) and the vertices are at (-1, 9) and (-1, -7)

The major axis is the vertical axis which is 2(8) = 16 and the minor axis is $12\sqrt{3}$.

References & Resources

Use these links for more information or come visit one of the FSU tutoring programs for one-on-one help!

- **Paul's Online Math Notes** (tutorial.math.lamar.edu/): The intent of this site is to provide a complete set of free online (and downloadable) notes and/or tutorials for math classes. They're written the notes/tutorials to be accessible to anyone wanting to learn the subject.

- **Trigonometry Review**: math.usask.ca/maclean/298/BW/trigreview.pdf

- **Desmos** (desmos.com/calculator): To create a new graph, just type your expression in the expression list bar. As you are typing your expression, the calculator will immediately draw your graph on the graph paper.

- **CalcPlot3D** (c3d.libretexts.org/CalcPlot3D/index.html): This dynamic Java applet allows the user to simultaneously plot multiple 3D surfaces, space curves, parametric surfaces, vector fields, contour plots, and more in a freely rotatable graph.

Tell us how we're doing!

The QR code below will take you to a survey about the Readiness Review Series.

We're always interested in improving, so we're asking for your feedback.

What was your experience like? Is there anything we should add, change, or remove?

Do you have ideas for how this program should be expanded in the future?

Scan the code and fill out the short survey to let us know!