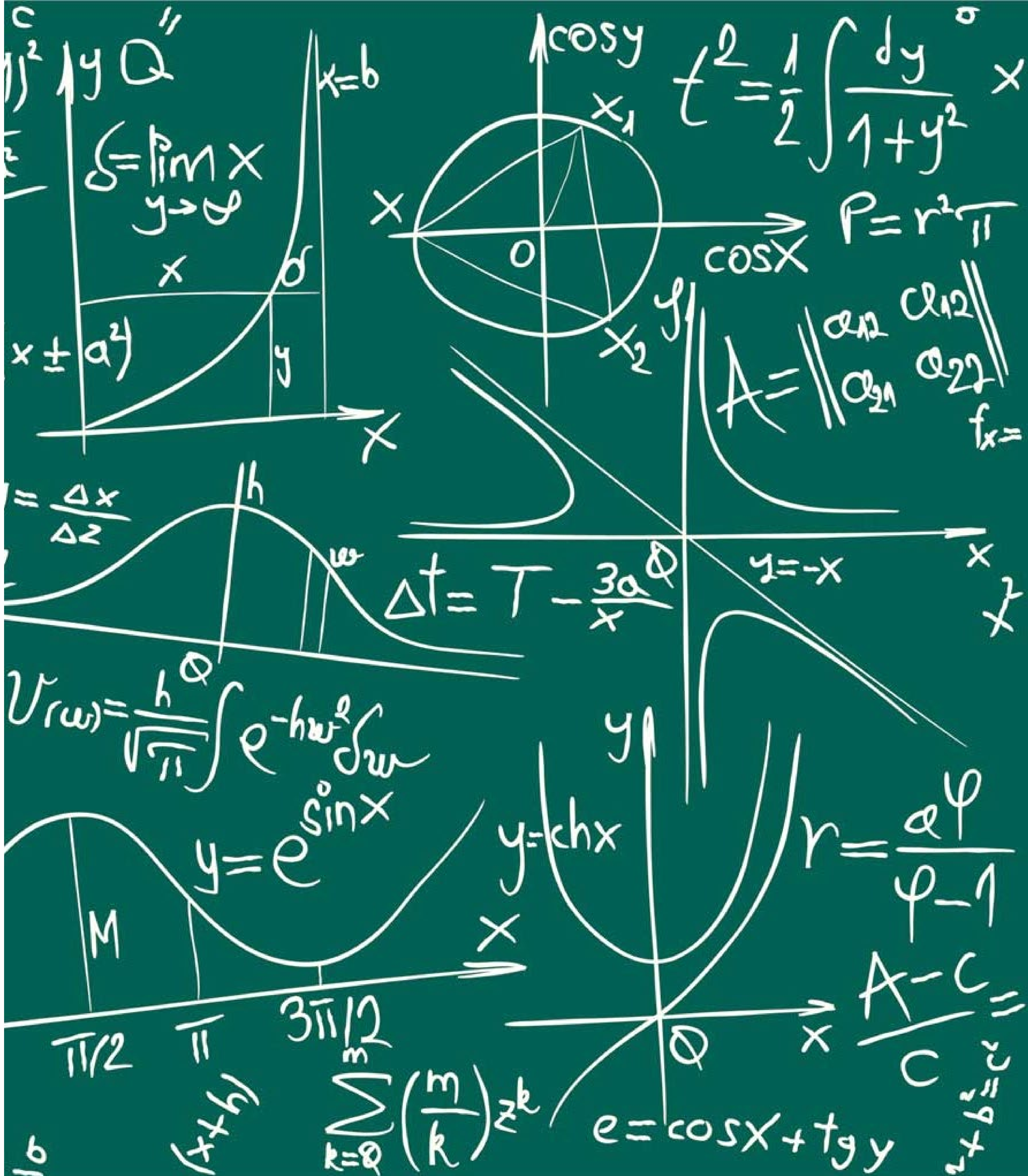
Readiness Review Series



tutoring.fsu.edu

Tips for Math Success

Believe that your intelligence can grow.

The first battle in learning anything is believing you're capable. Cultivating a growth mindset (i.e. believing you're fully capable of mastering new material) will help immeasurably in achieving academic success.

Practice by teaching someone else a difficult concept.

Explaining a concept to someone else will help you work through the material yourself and reinforce the information in your own mind. Once you can successfully teach someone else the concept, you know you've mastered it.

Get tutoring weekly.

ACE, CARE, and Libraries' Learning District all offer free help for math courses. Visit tutoring.fsu.edu for more information on what you can get help with and how to access those programs.

Memorize key formulas/theorums.

Having crucial formulas and concepts on hand will help you navigate the more complex concepts to come.

Tips for Math Success

Always attend class.

This is the best chance to hear extra explanations, ask questions, and gain a stronger understanding of how each concept fits into the overall subject matter.

Start homework the day it is assigned.

The best practice is to complete homework problems without using example problems as a guide or copying answers from another source. Also, even when it is not worth points, you should focus on mastering the content of these assignments.

Make your schedule work for you.

If you are taking 15 credits this semester, create a weekly study schedule with 25 hours of study time during the week. It's also better to create 30 minute time blocks per class throughout the week as opposed to cramming. You'll remember a lot more when exposed to the material multiple times by practicing problems 2 or 3 times outside of class each week.

Be an active reader.

When you read your textbook, paraphrase each paragraph or section to ensure you understand. It can also help to color code your notes to help you identify what you do not understand and give you the chance to ask for clarification later.

Tips for Math Success

Ask for help.

Spend a little bit of time trying to resolve it yourself, but don't spin your wheels.If something doesn't make sense or you feel stuck on a problem or concept, reach out to the instructor or the TAs for guidance. Visiting your instructors regularly in office hours will help you to develop better communication channels and to master the content you do not understand.

If you work with a tutor, make sure you have done some legwork before the tutoring session.

Make sure you know where you could use the additional help so that your tutoring session is effective and efficient.

Work well in advance of deadlines.

Last-minute emergencies and conflicts can never be predicted. You don't want to miss out on earning points because of procrastination.

Be comfortable being uncomfortable.

Learning takes time, and until we have mastered something, we may often lack confidence in our abilities and our knowledge. The more time you spend studying something, the more comfortable you will become with the topic. However, be patient with yourself as you are learning.

MAC2311 – Calculus I

Table of Contents

Vocabulary Review

Test your knowledge with these key terms.

Relationship between x(r), v(r), and a(r).

- 1st Derivative: $\frac{d}{dx}$ represents the slope of the tangent line on a graph, or a representation of the velocity at a particular instant. I.e., $\frac{d}{dx}y(x = a)$ would give you the velocity at position x = a.
- 2nd Derivative: Similarly, $\frac{d^2y}{dx^2}$ is the slope of the slope of the tangent line, or, the rate of change, of the rate of change, which is equivocally called the acceleration. Therefore a(r) = $\frac{d}{dx}$(v(r)) = $\frac{d^2y}{dx^2}$.

Integral/Integration: We learn about derivatives, really to just learn about integration, which is the most valuable math technique there is. Integrals can be defined in many ways. Their applications include finding the Areas, Volumes, Arc-length, Center of Mass, and anything in science that can be expressed as a summation, can be done as an integral, which is a lot of stuff.

Vocabulary Review

Test your knowledge with these key terms.

Rolle's → Mean Value Theorem:

- Suppose f(x) is a function that satisfies all of the following.

1. f(x) is continuous on the closed interval [a,b][a,b].
2. f(x) is differentiable on the open interval (a,b)(a,b).
3. f(a)=f(b)

- Then there is a number c such that a<c<b and f′(c) = 0. Or, in other words f(x) has a critical point in (a,b).

MVT:

- Suppose f(x)f(x) is a function that satisfies both of the following.

1. f(x)f(x) is continuous on the closed interval [a,b][a,b].
2. f(x)f(x) is differentiable on the open interval (a,b)(a,b).

- Then there is a number cc such that $a < c < b$ and

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

or

$$f(b) - f(a) = f'(c)(b - a)$$

Vocabulary Review

Test your knowledge with these key terms.

- Intermediate Value Theorem:
  - For any function f that's continuous over the interval [a,b] the function will take any value between f(a) and f(b) over the interval.
    - This also means “For any value L between f(a) and f(b), there exists a value c, within [a,b], such that f(c) = L

This is something that would intuitively make sense for any graph between two points that is continuous.

Limits

“Limits” describe how the function behaves when it gets close to the limit. They do not depend on the actual value of the function at the limit.

Let *f*(*x*) be a function defined on an interval that contains *x*=*a*, except possibly at *x*=*a*

The goal is to make f(x) as close to L as we want for all *x-values* that is close to *a*, without actually setting the x-value as *a*.

”The limit of...” ”...the function f...”

$$\lim_{x \to 3} f(x)$$

”...as x approaches 3.”

Limits: Method 1

Graphing

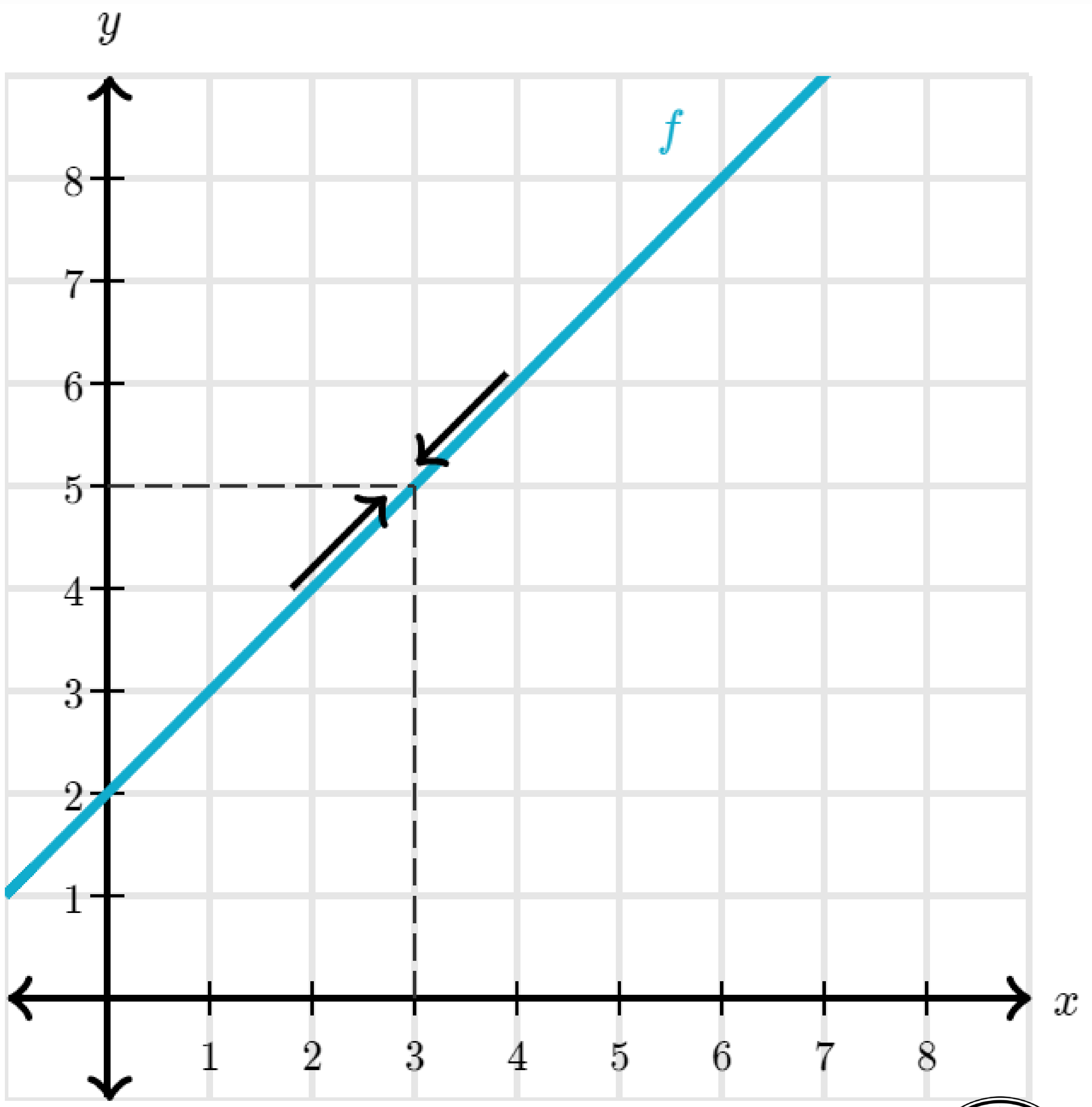
Graph $\lim_{x \to 3} f(x)$

$$f(x) = x + 2$$

The limit of f $at\ x = 3$ is the value f $(or\ the\ y - value)$ approaches y=5 as we get closer and closer to $x = 3$

Note: The $\lim_{x \to 3} f(x)$ =5 and the value of f(3)=5

*This does not always happen!



Limits: Method 2

Plugging in Values: Example 1

The goal in using limits is to get infinitely close to a specific y-value.

To find the $\lim_{x \to 3} f(x)$ where $f(x) = x + 2$ we could also plug in x-values smaller (or larger) than 3 and as those x-values come closer and closer to x=3, *f* becomes closer and closer to 5.

x	$f(x)$
2.9	4.9
2.99	4.99
$\underbrace{2.999}_{\text{close to 3}}$	$\underbrace{4.999}_{\text{close to 5}}$

x	$f(x)$
3.1	5.1
3.01	5.01
$\underbrace{3.001}_{\text{close to 3}}$	$\underbrace{5.001}_{\text{close to 5}}$

Limits: Method 2

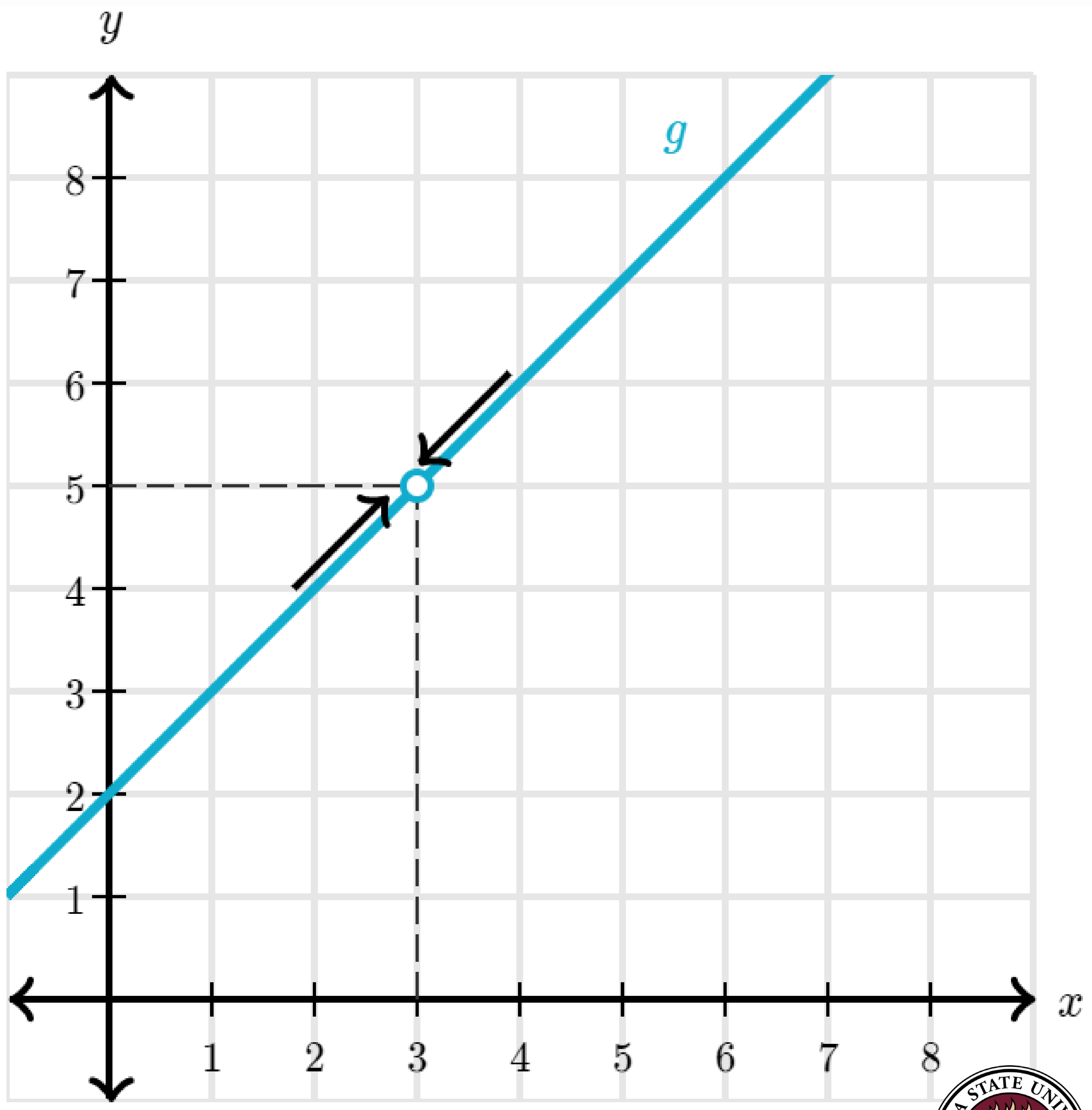
Plugging in Values: Example 2

Graph $\lim_{x \to 3} g(x)$

$$g(x) = \begin{cases} x + 2, & x \neq 3 \\ undefined, & x = 3 \end{cases}$$

The limit of g at $x = 3$ is $g = 5$ because we can get very very close to 5

Note: The $\lim_{x \to 3} g(x)$ = 5, but $g(3) = \text{undefined}$



Approaching Limits

From the Left and Right

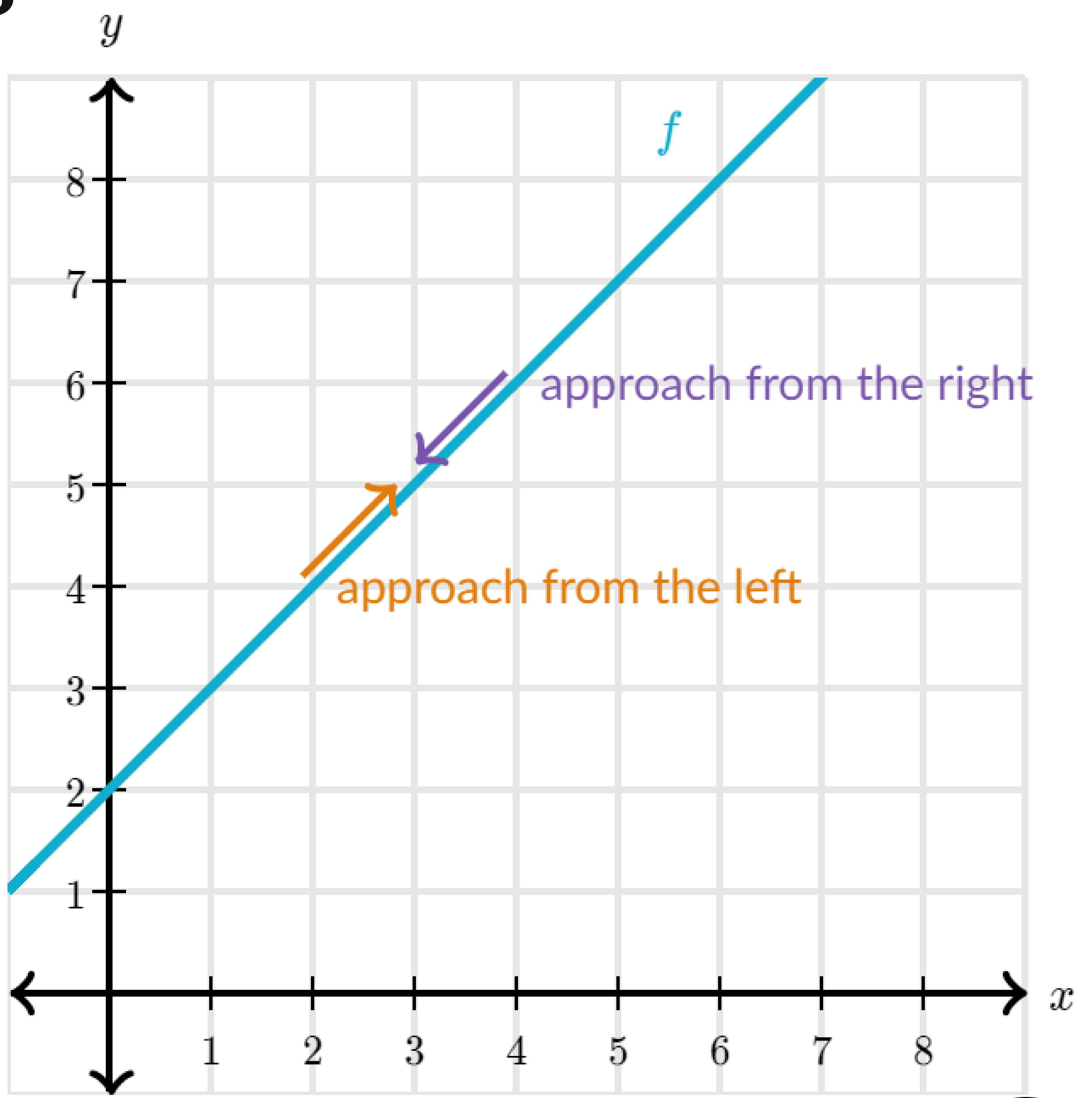
For a limit to be true, the answer (y-value) must be the same from both sides – whether you are approaching from the left or the right.

If the limit does not approach the same y-value from both sides then the limit does not exist (DNE).

The $\lim_{x \to 3} f(x)$ where $f(x) = x + 2$ shown from the left and the right:

Approaching from the left: $\lim_{x \to 3^-} f(x)$

Approaching from the right: $\lim_{x \to 3^+} f(x)$



Limit Discontinuities

Definitions and Examples

Definition: Limit discontinuities occur when the following is **NOT** true:

if *f(x)* is continuous @ x= c if and only if $\lim_{x \to c} f(x) = f(c)$

Discontinuities usually occur if you need to lift your pencil while drawing the graph of *f(x)*

Limit Discontinuities

Point/Removable

A graph is discontinued at a point and the limit may exist, but the function may not be defined at that point

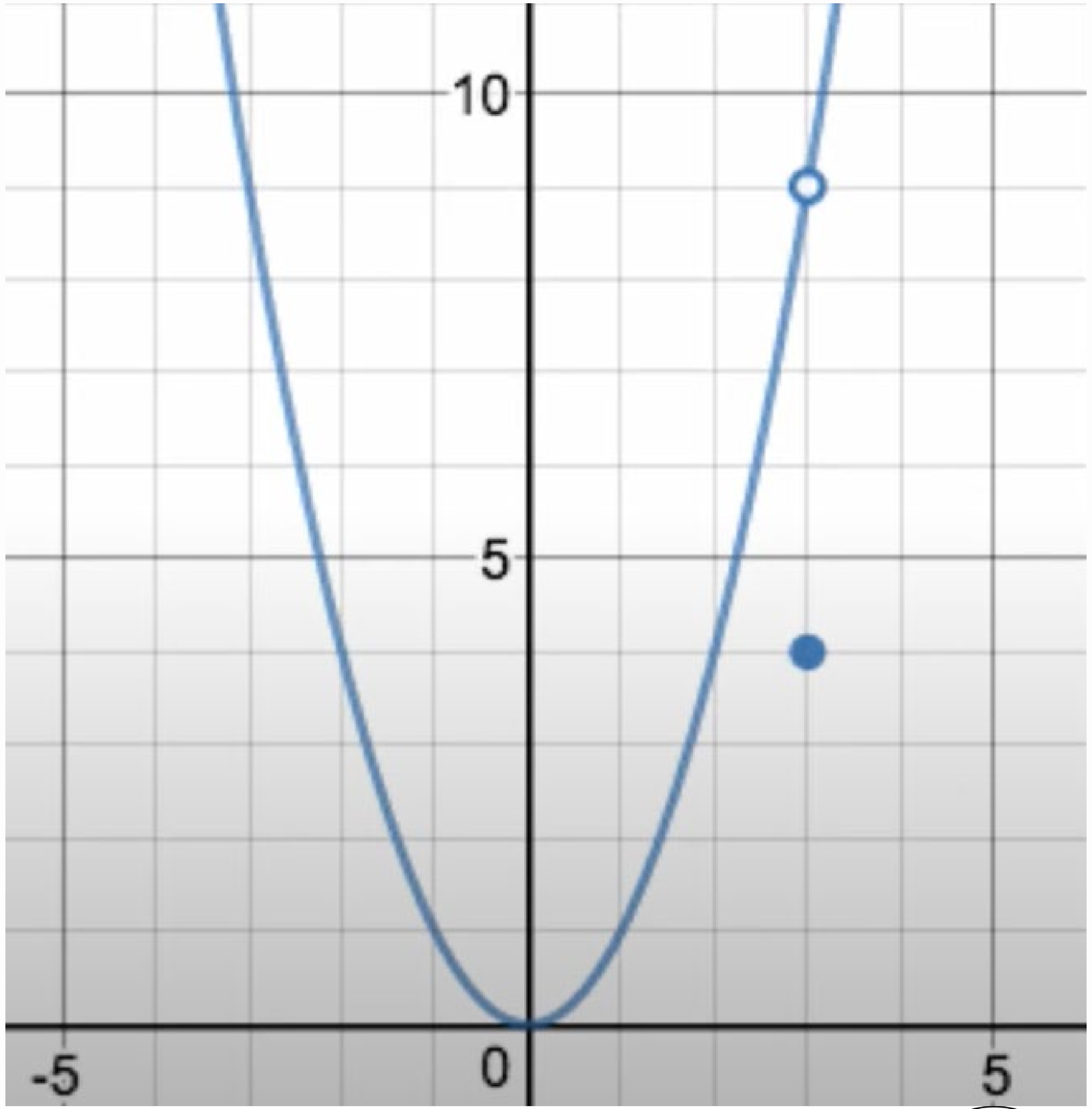
Example: $\lim_{x \to 3} f(x) = f(3)$ where $f(x) = x^2$

$$\lim_{x \to 3} f(x) = x^2 = 3^2 = 9$$

$f(3) = 4$ (found from the graph)

$\lim_{x \to 3} f(x) = 9$ does not equal $f(3) = 4$,

therefore, we say the limit exists, but is discontinuous at x=3



Limit Discontinuities

Jumps

When the left/right sided limits do not provide the same answer or when you must pick up your pencil to continue drawing the graph

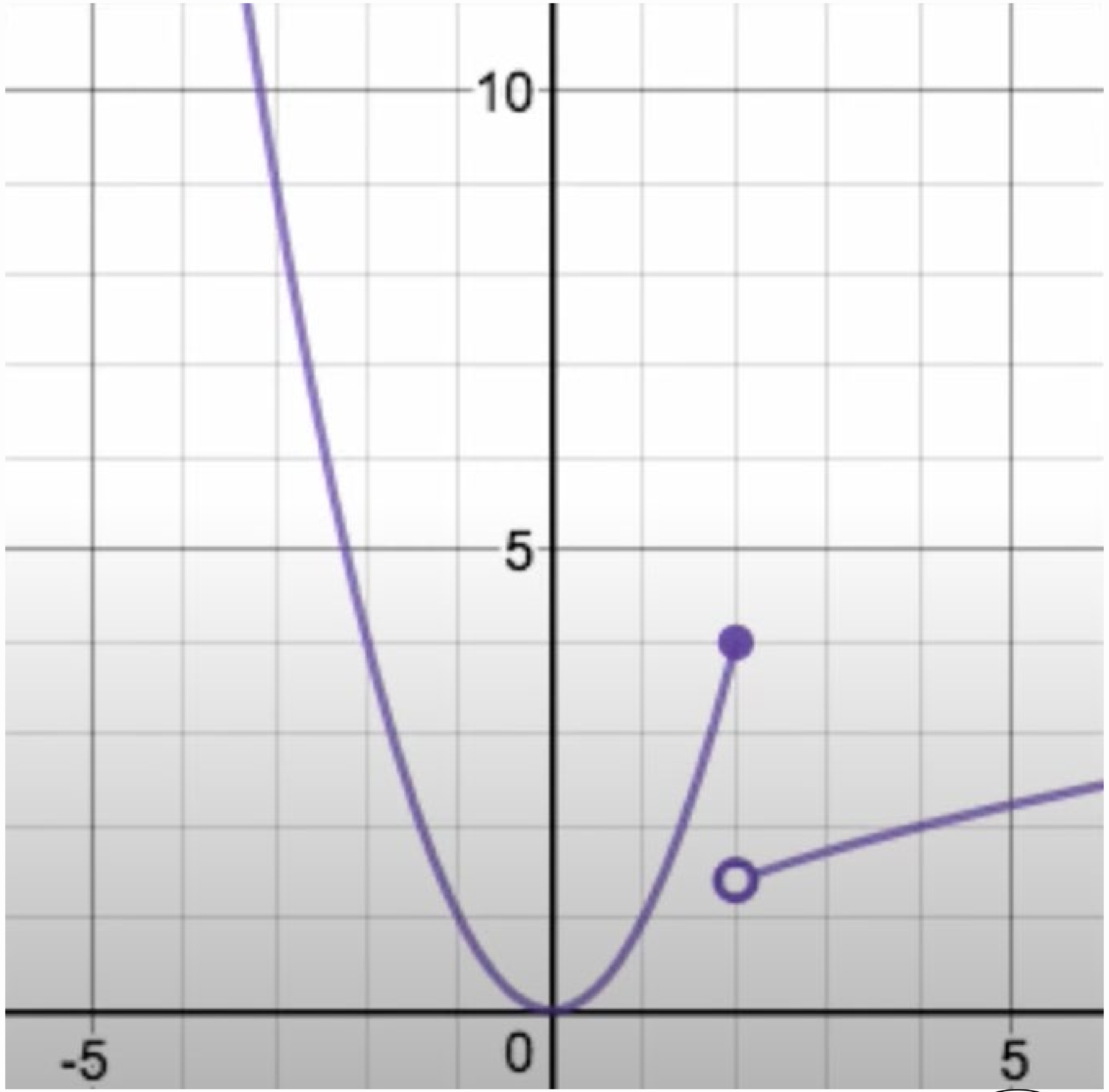
Using the graph of *f(x)*

$$\lim_{x \to 2^-} f(x) = 4$$

$$\lim_{x \to 2^+} f(x) = \sqrt[2]{2}$$

Since the limit from the left and the right are not equal to each other the limit does not exist and the continuity rule cannot be proven.

Rule: *f(x)* is continuous @ x= c if and only if $\lim_{x \to c} f(x) = f(c)$



Limit Discontinuities

Asymptotic

Graphs that have asymptotes

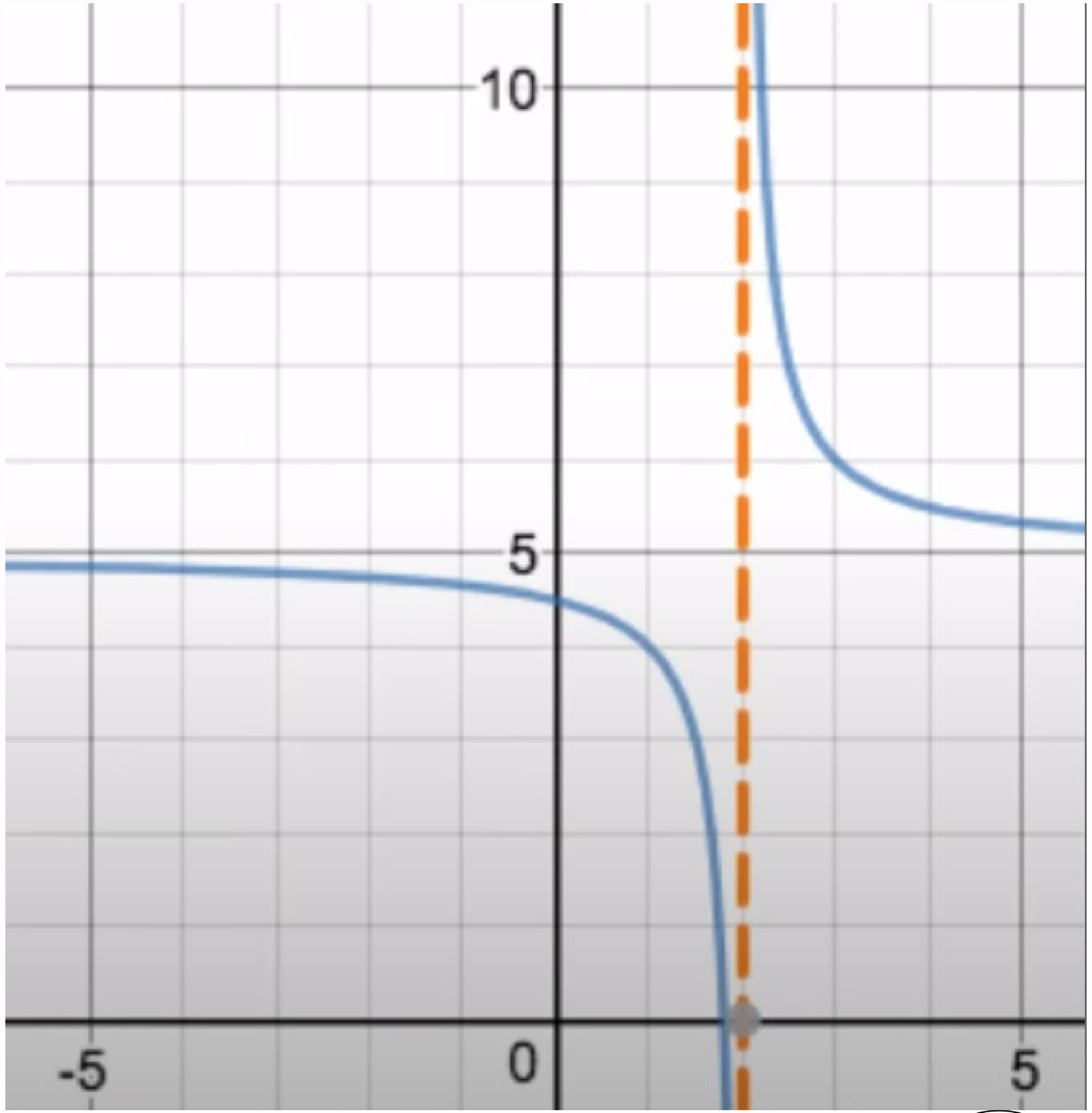
- The graph cannot be drawn without picking up your pencil

This type of discontinuity has unbounded limits as x approaches from the left and the right

Unbounded limits are those that go to infinity

$$\lim_{x \to 2^-} f(x) = -\infty$$

$$\lim_{x \to 2^+} f(x) = \infty$$



Infinite Limits

Uses and Context

Infinite limits can be used when functions approach a y-value at x=c where all or some of the graph continuously increases in y (positive infinity, ∞) or decreases in y (negative infinity, -∞).

The limit from the left and the right must be the same for this to be true

Infinite Limits

Example 1

Find $\lim_{x \to 0} \frac{1}{x^2}$ where $f(x) = \frac{1}{x^2}$

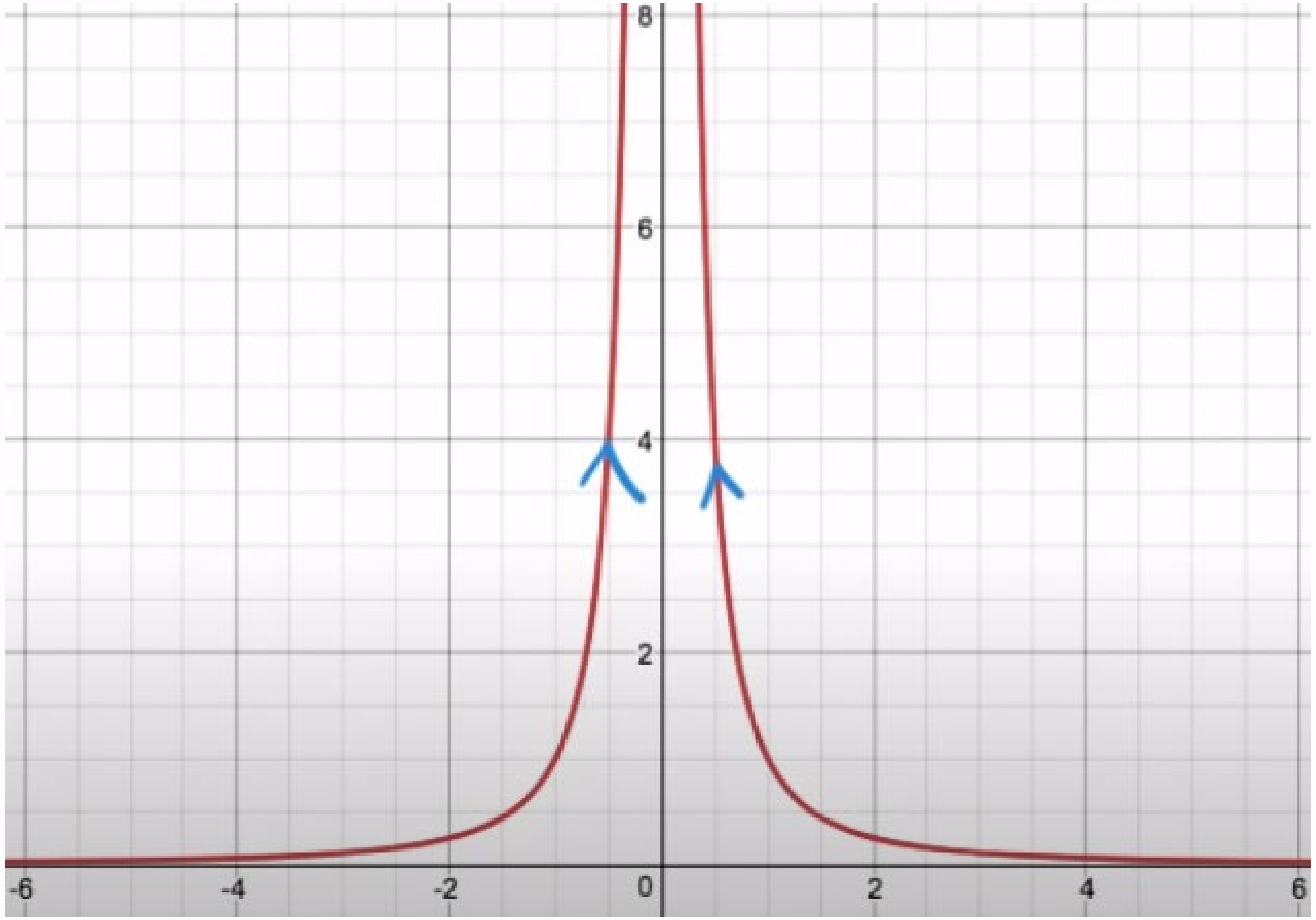
Graph f(x)

Determine the y-values of the limit from the left and from the right

$$\lim_{x \to 0^-} \frac{1}{x^2} = \infty$$

$$\lim_{x \to 0^+} \frac{1}{x^2} = \infty$$

The $\lim_{x \to 0} \frac{1}{x^2} = \infty$ because the limit to the left and the limit to the right are equal.



Infinite Limits

Example 2

Find $\lim_{x \to 0} \frac{1}{x}$ where $f(x) = \frac{1}{x}$

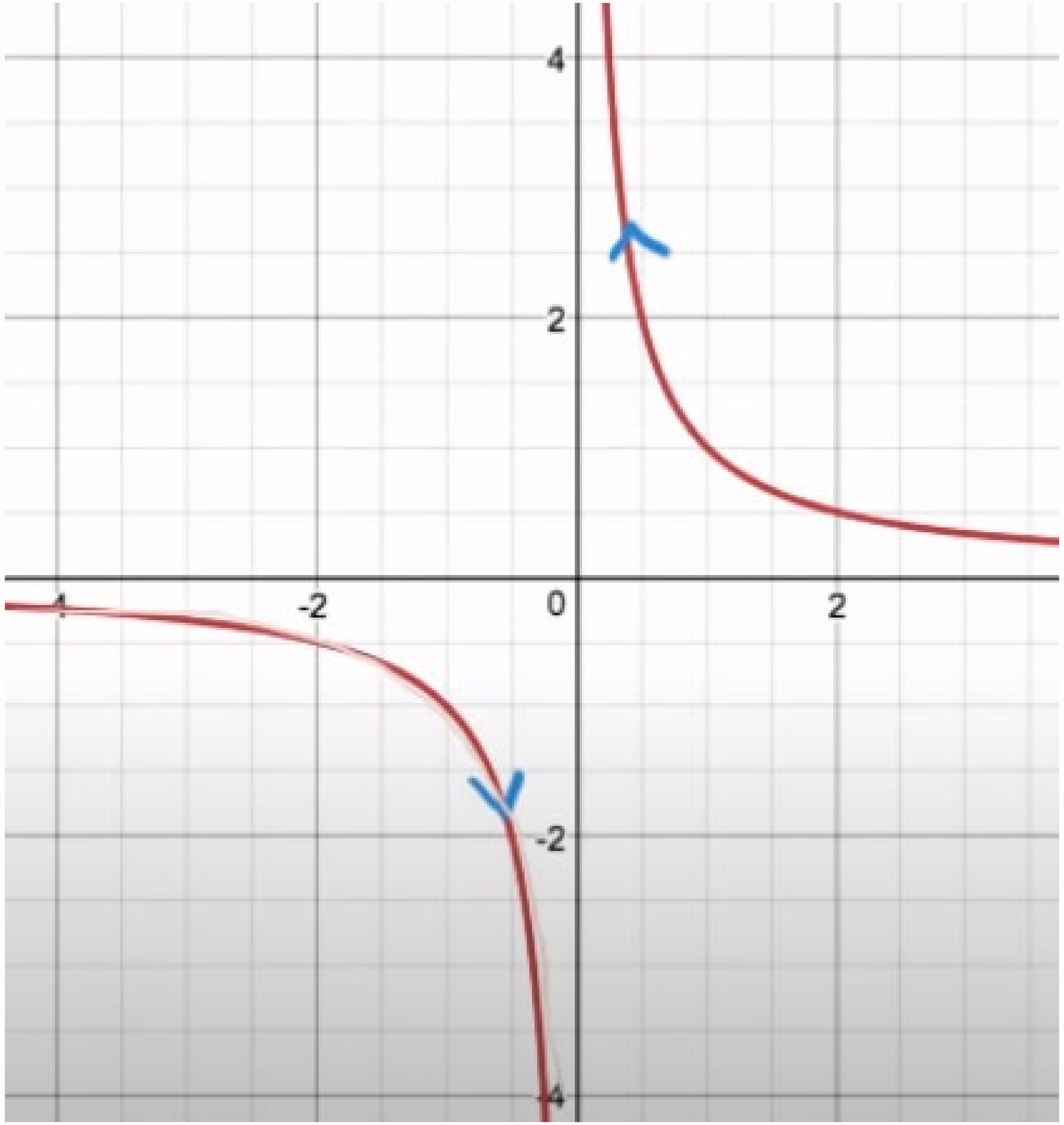
Graph *f(x)*

Determine the y-values of the limit from the left and from the right

$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$

$$\lim_{x \to 0^+} \frac{1}{x} = \infty$$

The $\lim_{x \to 0} \frac{1}{x} = DNE$ because the limit to the left and the limit to the right are **not** equal.



Limits at Infinity

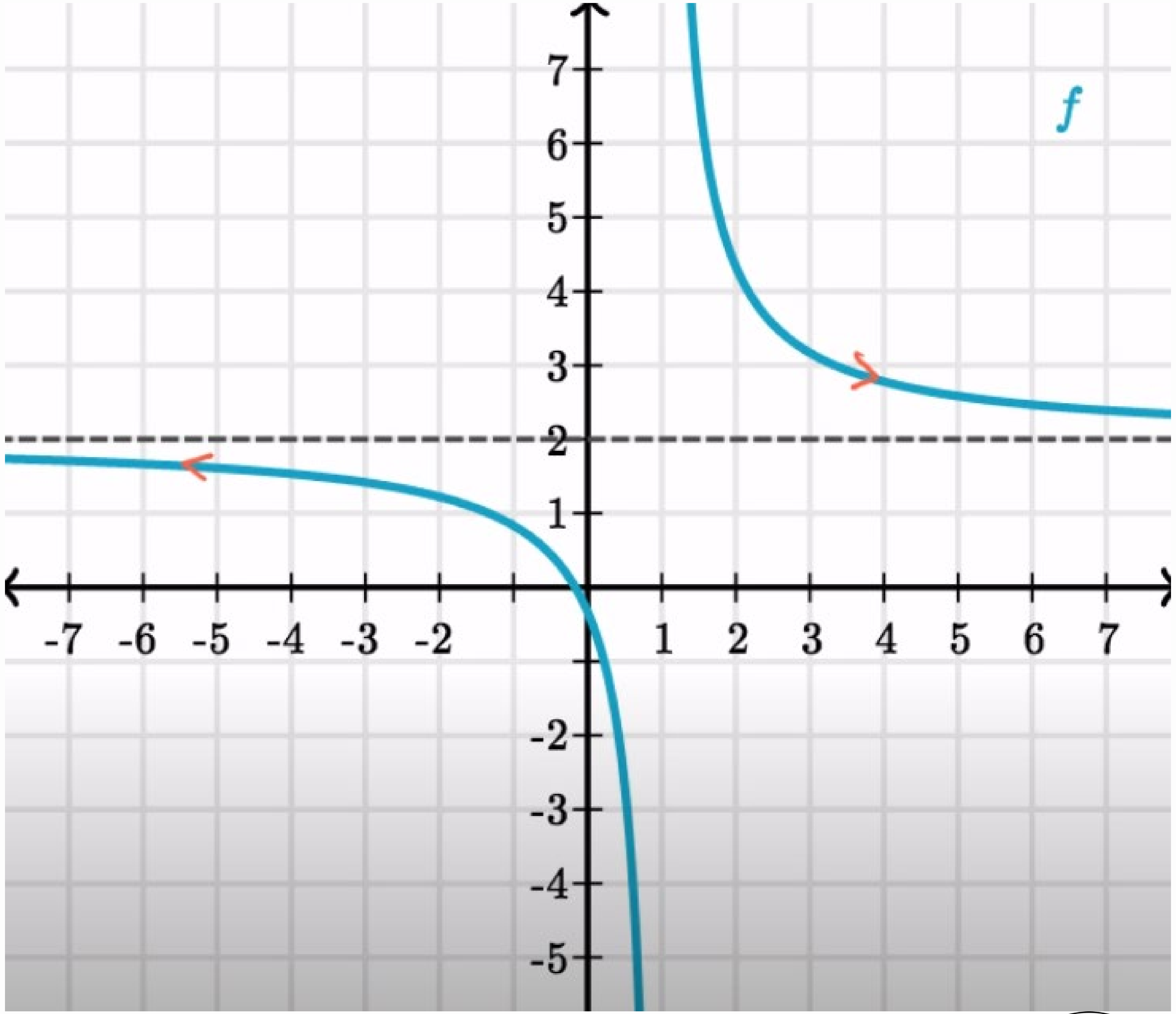
The limit of f of x as x approaches positive infinity is $\lim_{x\to\infty} f(x) = L$

<u>Example</u>: Use the graph find the limit of the function as x approaches infinity

Determine the y-values of the limit from the left and from the right

$$\lim_{x\to\infty} f(x) = 2$$
$$\lim_{x\to-\infty} f(x) = 2$$

The $\lim_{x\to\infty} f(x) = 2$ because the limit to the left and the limit to the right **are** equal.



Derivative & Integration Techniques

- **Power Rule:** $\frac{d}{dx}(x)^n = n(x)^{(n-1)}$
- **Product Rule:** $\frac{d}{dx}\big(F(x)G(x)\big) = F'(x)S(x) + F(x)S'(x)$
    - Think F = first, and S = second, and both functions must be with respect to x.
- **Chain Rule:** $\frac{d}{dx}\big(f\big(g(x)\big) = f'\big(g(x)\big)g'(x) = n\big(g(x)\big)^{n-1} * g'(x)$
- **Trig:** $\frac{d}{dx}\sin(x) = \cos(x)$ $\frac{d}{dx}\cos(x) = -\sin(x)$ …

  $\frac{d}{dx}tan(x) = sec^2(x)$ $\frac{d}{dx}\sec(x) = sec(x)\tan(x)$
- **Integration**: U-substitution, Trig-substitution

$$\int (x+1)^2 dx, \quad let\ u=x+1,\ \therefore\ dx=du$$
$$=\int u^2 du$$
$$=\frac{1}{3}u^3+C$$
$$=\frac{(x+1)^3}{3}+C$$

$$\int_0^1 \sqrt{1-x^2}\,dx = \int_0^{\pi/2} \sqrt{1-\sin^2 u}\cos(u)\,du$$
$$= \int_0^{\pi/2} \cos^2 u\,du$$
$$= \left[\frac{u}{2}+\frac{\sin(2u)}{4}\right]\Bigg|_0^{\pi/2}$$
$$= \frac{\pi}{4}+0=\frac{\pi}{4}.$$

More Integration Techniques

Most Common Methods/Integrals (That you should know easily)

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad \text{if } n \neq -1$$

$$\int x^{-1}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

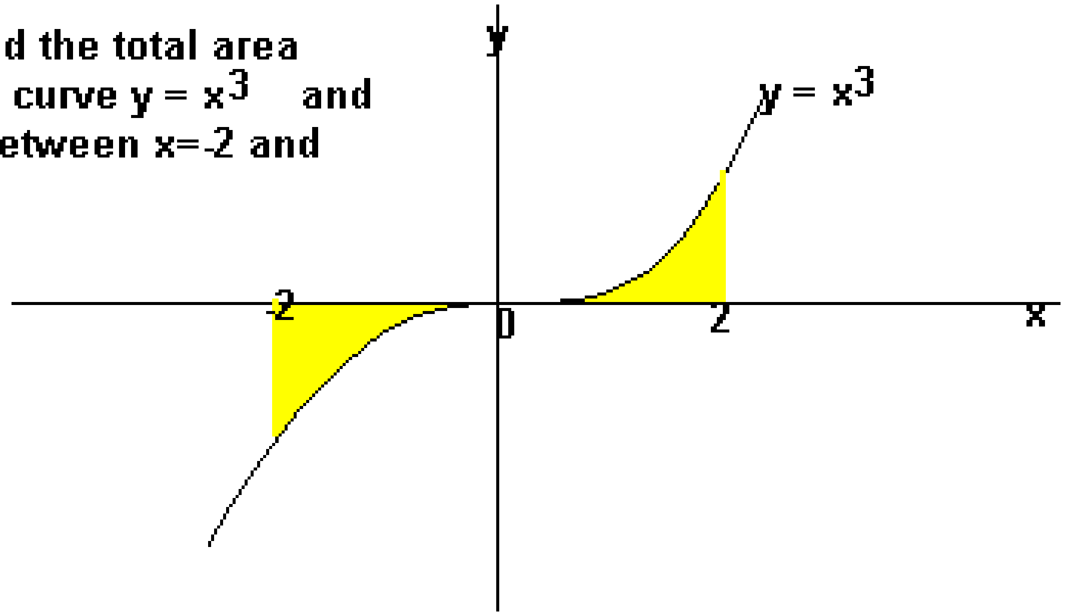
$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \frac{1}{1+x^2}\,dx = \arctan x + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$$

Example: find the total area between the curve $y = x^3$ and the x-axis between x=-2 and x=2.



If we simply integrated x^3 between -2 and 2, we would get:

$$\left[\frac{x^4}{4}\right]_{-2}^{2} = 4 - 4 = 0$$

So instead, we have to split the graph up and do two separate integrals.

$$\int_0^2 x^3\,dx = \left[\frac{x^4}{4}\right]_0^2 = 16/4 - 0 = 4$$

$$\int_{-2}^0 x^3\,dx = \left[\frac{x^4}{4}\right]_{-2}^0 = 0 - 16/4 = -4 \qquad \text{(so area is 4).}$$

We then add these two up to get: 8 units^2

References & Resources

Use these links for more information or come visit one of the FSU tutoring programs for one-on-one help!

- **Paul's Online Math Notes** (tutorial.math.lamar.edu/): The intent of this site is to provide a complete set of free online (and downloadable) notes and/or tutorials for math classes. They're written the notes/tutorials to be accessible to anyone wanting to learn the subject.

- **Desmos** (desmos.com/calculator): To create a new graph, just type your expression in the expression list bar. As you are typing your expression, the calculator will immediately draw your graph on the graph paper.

- **CalcPlot3D** (c3d.libretexts.org/CalcPlot3D/index.html): This dynamic Java applet allows the user to simultaneously plot multiple 3D surfaces, space curves, parametric surfaces, vector fields, contour plots, and more in a freely rotatable graph.

References & Resources

Other Useful Websites, Reviews, and Sources:

- https://www.khanacademy.org/math/calculus-1/cs1-limits-and-continuity/cs1-limits-intro/a/limits-intro?modal=1
- https://www.khanacademy.org/math/calculus-1/cs1-limits-and-continuity
- https://math.libretexts.org/Bookshelves/Calculus/Book%3A_Calculus_(OpenStax)/06%3A_Applications_of_Integration
- https://ocw.mit.edu/resources/res-18-001-calculus-online-textbook-spring-2005/study-guide/
- https://sites.math.washington.edu/~aloveles/Math126Spring2018/Calc1Review.pdf

Tell us how we're doing!

The QR code below will take you to a survey about the Readiness Review Series.

We're always interested in improving, so we're asking for your feedback.

What was your experience like? Is there anything we should add, change, or remove?

Do you have ideas for how this program should be expanded in the future?

Scan the code and fill out the short survey to let us know!